Calculus Expectations and Retention

Ed Wheeler
University of Tennessee at Martin, ewheeler@utm.edu

Abstract - **Student retention is an important key to sustaining a successful engineering program. Without a supply of students that are capable of completing the challenges of the typical engineering curriculum, engineering programs would soon disappear or be required to reduce the rigor of the curriculum. Typically the undergraduate engineering student that successfully completes the calculus series is well on his/her way to completion of the degree requirements. This paper will examine the skill sets that are associated with ACT math sub-scores and the required skill sets that are required for a typical first-semester calculus class. Preliminary investigation in this area at The University of Tennessee at Martin indicates that inconsistencies exist between the two skill sets. Determining whether or not consistency exists between the two skill sets will provide an insight into the possible disconnect between what the student is capable of performing and what the student is expected to perform in the first-semester calculus class. With this information admissions requirements, curriculum adjustments, etc. can be proposed resulting in higher passing rates in calculus, thus higher retention rates in engineering.**

Index Terms - Retention, Calculus expectations, Calculus readiness.

INTRODUCTION

Bernstein, et al concluded that the number of engineering graduates since the late 1990's has been sufficient to meet the demands of industry [1]. Unfortunately for the industries in need of new engineers many of the top engineering graduates were lured to careers in finance and consulting. As engineering graduates continue to move to more lucrative fields the need for graduates will continue to increase.

The Bureau of Labor Statistics Occupational Outlook Handbook, 2010-11 states that employment of engineers is expected to grow by 11 percent over the 2008-18 decade. Job opportunities for biomedical engineers are expected to increase the most with a growth rate of 72% over the decade. Civil engineering jobs are projected to have the largest growth in number with employment increasing by 67,600 positions [2]. The challenge for schools of engineering in the United States will be to continue to meet the demand for these positions. Current retention rates of 50-55% must be maintained or improved in order to meet these demands [3].

RETENTION

Retention is a complex issue that can be defined in many different manners. Retention issues and efforts are typically tailored to specific universities in order to be program specific. Analysis of loss of engineering students must address a number of issues including interdepartmental transfers, transfers to other colleges at a university, transfers to another university, abandonment of interest in university education and transfers into engineering from within the university or from other universities [4]. For the purpose of this paper retention will be defined as first-year academic success and retention.

A key theme which emerged from a National Science Board Workshop, *Moving Forward to Improve Engineering Education*, was retention of engineering students, especially in the first year, must be approached as a systems problem. When considering the retention issue the student supply pipeline as well as the cultural perceptions of engineering and engineers must be addressed. Pre-college preparation of entering students affects retention, as does the difficulty of the engineering curriculum. The difficulty of the engineering curriculum relative to other academic majors and the perceived relative value of the engineering degree must also be taken into consideration [5].

Retention Models

The student attrition away from engineering has been described with numerous models. Johnson and Shepard view the situation as a pipeline. This model envisions a leaky pipeline with the leaks representing student attrition from middle school thru graduation from an engineering college [6]. Adelman argues that the correct model is a path model not a pipeline model. The freshman courses in engineering include chemistry, physics and math, allowing students to switch to a science major with little loss of coursework. He views the choice of major as a competitive one among several choices (paths) [7]. A very common competing interest (path) is business, which students often choose when leaving the field of engineering. Watson and Froyd see the model as a transmission line with three component lines. The three component lines are cognitive ability development, occupational choice development and self-identity development. Similar to transmission lines each of the parts is viewed as having an interference field associated with it. One field could affect the transmission of another line if one of the areas is not sufficiently developed. For example, if a student must spend a great deal of energy in establishing his/her self-identity in the engineering college, it may influence cognitive ability development affecting his/her success in first term courses [8].

As discussed by Dey, et al Tinto's Interactionalist Theory of Departure Model presents a model of new student adjustment to college. The model theorizes that the pre-college characteristics of family background, skills and abilities and prior schooling are important predictors of success in college. Family background includes social status, the education of the parents and community size. Skills and abilities include intellectual and social skills, financial resources, motivations and political preferences. Prior schooling includes educational preparation and experiences [9].

According to Tinto, a student transitions from the familial environment and adjusts to college culture. As this adjustment to college occurs, a process of both academic and social integration is required for the successful adjustment of the student. Academic integration is broadly defined as doing well in courses and social integration includes the development of social relationships with other students and with faculty. As academic and social integration occurs, a student reaches a new level of learning. This level of learning translates into value-added education, student success and potential persistence. Each student comes to college with a set of career and college goals. As integration occurs, the student may change his/her goals for college with respect to a major or career. This can lead to the decision to change major, remain enrolled in or depart from the institution [9].

Using Tinto's Model as a platform Dey, et al developed a transitional model specifically targeted for the retention of engineering students. The model, while similar to Tinto's model, includes additional pre-college characteristics important for engineering student success. These characteristics were grouped into nine categories presented in Table I.

TABLE I
PRE-COLLEGE CHARACTERISTICS IMPORTANT FOR ENGINEERING STUDENT ACADEMIC SUCCESS

Pre-College Characteristic	Typical Indicator
High School Academic Achievement	High School GPA of High School Rank; ACT Composite or SAT Total
Quantitative Skills	ACT Math or SAT Math
Study Habits	Hour/week studied in high school
Commitment to Career and Educational Goals	Expected degree and career
Confidence in Quantitative Skills	Self-rating of confidence in math, science and computers
Commitment to Enrolled College	Choice of college, reason for choosing this college, satisfaction with choosing this college
Financial Needs	Amount of loans, percent of financial needs that are not met
Family Support	Educational level of parents, Income level of parents
Social Engagement	Social involvement; connections with teachers and other students

Source Dey, et al [9]

The Engineer of 2020 published by the National Academy of Engineering proposed nine key attributes of an engineer. Dey, et al summarized these attributes by stating "an engineering student is preparing for a career as an analytical thinker who can lead people in technology innovation, design and systems thinking. From the college curriculum, the courses most strongly related to analytical thinking are mathematics and science courses. The engineering freshman curriculum is weighted with mathematics and science courses" [9].

Based upon a review of empirical studies on student retention performed by Dey, et al a significant difference between the pre-college characteristics for engineering education studies and general education studies exists. The defining difference is the need for quantitative skills. The sub-score obtained on the ACT or SAT was determined to be a significant predictor for academic success as measured by GPA and retention. This is consistent with the conclusion drawn in *The Engineer of 2020*. A strong analytical background is required to successfully navigate the engineering curriculum. Not only is the presence of quantitative skills critical to success but confidence in these skills is also required [9].

A MathSoft survey of engineering professors found that 43% thought the reason for a student dropping engineering as a major was difficulty understanding math [10]. Based on this perception and the conclusions drawn by Dey, et al it is imperative that students be placed in the correct mathematics course upon entering a college to pursue an engineering degree.

MATH PLACEMENT

Recall the often told story of the engineering dean who upon greeting the incoming freshman class tells the students to look to their left and look to their right and then proceeds to announce that two from each group of three will not be an engineering major by the end of the year. The shortage of engineers in the 1980's brought about a lasting change in this attitude. Schools of engineering can ill afford to lose two-thirds of their entering freshman class. No longer can the entering first-year student be "thrown into the pool" to sink or swim.

Not every student with the intention of becoming an engineer will have sufficient pre-college quantitative skills to master calculus in the first semester of college. This is not to suggest those students who lack these skills are to be relegated to another major. Rather the contrary applies; these students should be allowed to develop the quantitative skills before attempting calculus. This is where proper math placement enters into the retention equation.

Universities such as The Ohio State University, The University of Texas, Virginia Tech and Illinois Urbana-Champaign make use of placement exams to assess the quantitative abilities of entering students. The results of these exams are used to place students into the proper math courses. Researching the websites of major universities finds ALEKS to be a common source of tests used for math placement. ALEKS (Assessment and LEarning in Knowledge Spaces) is a web based assessment and learning technology developed by cognitive scientists and software engineers at the University of California, Irvine [11].

For example the University Of Texas Cockrell School Of Engineering requires all freshmen applicants to provide one of the following to qualify for college-level calculus.

- A score of at least 560 on the SAT Subject Test in Mathematics: Level 1 exam or a score of at least 530 on the SAT Subject Test in Mathematics: Level 2 exam.
- An official AP Calculus exam score of 3 or higher by the end of junior year.
- A transcript showing college credit with a grade of C or higher in a pre-calculus or calculus course.
- A score of at least 80 on ALEKS.

Applicants who do not provide the required proof of calculus readiness by the official admissions application deadline can continue through the admissions process by submitting an SAT Math score of at least 600, an ACT Math score of at least 26, or documentation of concurrent enrollment in dual credit pre-calculus or calculus class [12].

Web research indicates a range of 24 to 30 ACT math sub-score requirements to qualify for enrollment in first semester calculus at universities not making use of placement exams. The wide range of scores indicates a greatly varying opinion of the skill set required for the first semester calculus course. ACT associates a particular skill set with scores of 24 to 27 and another set with scores of 28 to 32. Table II summarizes the skill set for scores from 24 to 36.

The University of Tennessee at Martin (UTM) Department of Mathematics and Statistics requires a student to successfully complete (defined as a grade of C or better) a five hour pre-calculus course or enter the University with an ACT math sub-score of 27 or higher before enrolling in the first semester of calculus. Approximately 35-40% of the freshmen engineering students qualify for calculus based on ACT math sub-score.

The UTM Office of Institutional Research periodically provides performance information to the state higher education board. In the process of collection and analysis of the data, skill sets required for all college level mathematics courses were determined in cooperation with the Department of Mathematics and Statistics. The desired ACT readiness skills required for successful completion of the first calculus course are indicated by italics in Table II.

Examination of the table reveals expectations that are not aligned with minimum score requirements. Forty-five skills identified as necessary for successful completion of the calculus class are skills associated with ACT math scores higher than the minimum score required for enrollment. This presents the conundrum for math placement. Math departments expect higher level skills than indicated by the ACT.

TABLE II
SKILLS ASSOCIATED WITH MATH ACT SUB-SCORE

	ACT MATH SUB-SCORE		
	24-27	28-32	33-36
Basic Operations & Applications	*Solve multistep arithmetic problems that involve planning or converting units of measure*	*Solve word problems containing several rates, proportions, or percentages*	*Solve complex arithmetic problems involving percent of increase or decrease and problems requiring integration of several concepts from pre-algebra and/or pre-geometry*
Probability, Statistics, & Data Analysis	*Calculate the average, given the frequency counts of all the data values* *Manipulate data from tables and graphs* Compute straightforward probabilities for common situations Use Venn diagrams in counting	*Calculate or use a weighted average* *Interpret and use information from figures, tables, and graphs* Apply counting techniques Compute a probability when the event and/or sample space are not given or obvious	Distinguish between mean, median, and mode for a list of numbers *Analyze and draw conclusions based on information from figures, tables, and graphs* Exhibit knowledge of conditional and joint probability
Graphical Representations	*Identify the graph of a linear inequality on the number line* *Determine the slope of a line from points or equations* *Match linear graphs with their equations* *Find the midpoint of a line segment*	*Interpret and use information from graphs in the coordinate plane* *Match number line graphs with solution sets of linear inequalities* *Use the distance formula* *Use properties of parallel and perpendicular lines to determine an equation of a line or coordinates of a point* *Recognize special characteristics of parabolas and circles*	*Match number line graphs with solution sets of simple quadratic inequalities* *Identify characteristics of graphs based on a set of conditions or on a general equation such as* $y = ax^2 + c$ *Solve problems integrating multiple algebraic and/or geometric concepts* *Analyze and draw conclusions based on information from graphs in the coordinate plane*

	ACT MATH SUB-SCORE		
	24-27	*28-32*	*33-36*
Properties of Plane Figures	*Use several angle properties to find an unknown angle measure* *Recognize Pythagorean triples* *Use properties of isosceles triangles*	*Apply properties of 30°-60°-90°, 45°-45°-90°, similar, and congruent triangles* *Use the Pythagorean theorem*	*Draw conclusions based on a set of conditions* *Solve multistep geometry problems that involve integrating concepts, planning, visualization, and/or making connections with other content areas* *Use relationships among angles, arcs, and distances in a circle*
Numbers: Concepts & Properties	*Find and use the least common multiple* *Order fractions* *Work with numerical factors* *Work with scientific notation* *Work with squares and square roots of numbers* *Work problems involving positive integer exponents* *Work with cubes and cube roots of numbers* *Determine when an expression is undefined* *Exhibit some knowledge of the complex numbers*	*Apply number properties involving prime factorization* *Apply number properties involving even/odd numbers and factors/multiples* *Apply number properties involving positive/negative numbers* *Apply rules of exponents* *Multiply two complex numbers*	*Draw conclusions based on number concepts, algebraic properties, and/or relationships between expressions and numbers* *Exhibit knowledge of logarithms and geometric sequences* *Apply properties of complex numbers*
Expressions, Equations, & Inequalities	*Solve real-world problems using first-degree equations* *Write expressions, equations, or inequalities with a single variable for common pre-algebra settings* *Identify solutions to simple quadratic equations* *Add, subtract, and multiply polynomials* *Factor simple quadratics* *Solve first-degree inequalities that do not require reversing the inequality sign*	*Manipulate expressions and equations* *Write expressions, equations, and inequalities for common algebra settings* *Solve linear inequalities that require reversing the inequality sign* *Solve absolute value equations* *Solve quadratic equations* *Find solutions to systems of linear equations*	*Write expressions that require planning and/or manipulating to accurately model a situation* *Write equations and inequalities that require planning, manipulating, and/or solving* *Solve simple absolute value inequalities*
Measurement	*Compute the area of triangles and rectangles when one or more additional simple steps are required* *Compute the area and circumference of circles after identifying necessary information* *Compute the perimeter of simple composite geometric figures with unknown side lengths*	*Use relationships involving area, perimeter, and volume of geometric figures to compute another measure*	*Use scale factors to determine the magnitude of a size change* *Compute the area of composite geometric figures when planning or visualization is required*
Functions	*Evaluate polynomial functions, expressed in function notation, at integer values* *Express the sine, cosine, and tangent of an angle in a right triangle as a ratio of given side lengths*	*Evaluate composite functions at integer values* *Apply basic trigonometric ratios to solve right-triangle problems*	*Write an expression for the composite of two simple functions* *Use trigonometric concepts and basic identities to solve problems* *Exhibit knowledge of unit circle trigonometry* *Match graphs of basic trigonometric functions with their equations*

Source: ACT [13] and the UTM Office of Institutional Research and Planning

Using data collected by the UTM Department of Mathematics and Statistics dating from 1999-2008 it was determined 94 first-year students out of a sample of 265 first-year students were not successful in first-semester calculus when placed according to the ACT math sub-score. This represents 35.5% of all first-year, first-time takers. The overall grade distribution and average ACT math sub-score for the letter grade is summarized in Table IV.

TABLE IV
FIRST-YEAR CALCULUS GRADE DISTRIBUTION
AND AVERAGE MATH SUB-SCORE

Letter Grade	Percentage of Assigned Grades	ACT Math Sub-score Average
A	25.3	29.6
B	21.5	29.0
C	17.7	28.3
Below C	35.5	28.4

The average appears to indicate the higher the ACT math sub-score the more likely the student will receive a successful grade. It should be mentioned that in the data examined 17 students with sub-scores of 30 or greater were not successful in the course. The data is not available on the breakdown of students who persisted to the end of the course and students who "stopped-out" or quit attending.

CONCLUSIONS

Fifty to fifty-five percent of the students who begin college majoring in engineering complete their program. Of the students who leave engineering, approximately half of the students drop out after the first year [9]. Based on a survey of engineering professors over 40% of those who drop out do so because of math difficulties [10]. Could better math placement result in fewer students dropping out? The evidence suggests students do not have the skills necessary for calculus based on ACT math sub-scores of 27-30. Comparison of skills required and skills identified by the ACT indicates scores of 33-36 are required to be successful. It is unrealistic to set the gateway score at 33-36 for entrance into the first calculus class. A more realistic score is 29 based on the grade distribution presented.

The research presented here tends to indicate inconsistencies in skills required for calculus and the skill identified by the ACT math sub-scores. This can have a major impact on the retention of students required to take calculus. Students placed in calculus without the readiness skills for the class will become frustrated and drop the class or if they persist through the class they will not be successful. This leads to loss of confidence in quantitative skills and possibly a change in major. One solution to the placement conundrum is to use placement exams and end reliance on the ACT scores. The goal of any solution should be to place the student in a mathematics course where he/she will have the highest probability of success; even if it means delaying entrance into calculus and the engineering curriculum.

REFERENCES

[1] B. Lindsay Lowell, B. L., Salzman, H., Bernstein, H., Henderson, E., "Steady as She Goes? Three Generations of Students through the Science and Engineering Pipeline" *Proceedings of the Annual Meetings of the Association for Public Policy Analysis and Management* Washington, D.C. November 7, 2009, http://www.heldrich.rutgers.edu/uploadedFiles/Publications/STEM_Paper_Final.pdf.

[2] "Occupational Outlook Handbook, 2010-11 Edition", Bureau of Labor Statistics, United States Department of Labor, http://www.bls.gov/oco/ocos027.htm.

[3] Wulf, W. A., Fisher G. M. C., "A Makeover for Engineering Education**",** *Issues in Science and Technology*, Spring, 2002, www.issues.org/18.3/p_wulf.html.

[4] "Undergraduate Engineering Retention as Measured by Degree and Enrollment Comparisons - Data for the Last Half Century", *Engineering Trends*, February, 2006, http://www.engtrends.com/IEE/0206A.php.

[5] "Summary notes: Moving Forward to Improve Engineering Education", *National Science Board*, November 7, 2006, http://www.nsf.gov/pubs/2007/nsb07122/nsb07122_4.pdf.

[6] Johnson, M. J., Sheppard S. D. "Students Entering and Exiting the Engineering Pipeline-Identifying Key Decision Points and Trends." *2002 Proceedings of 32nd ASEE /IEEE Frontiers in Education Conference*, Session S3C, http://fie-conference.org/fie2002/index.htm.

[7] Adelman, C., "Women and Men of the Engineering Path: A Model for Analyses of Undergraduate Careers", Office of Educational Research and Improvement, U.S. Department of Education, 1998, http://www.eric.ed.gov/ERICDocs/data/ericdocs2sql/content_storage_01/0000019b/80/15/7d/35.pdf

[8] Watson, K., Froyd J., "Diversifying the U.S. Engineering Workforce: A New Model", *Journal of Engineering Education*, 96, 1, 2007, 19-32, http://www.asee.org/publications/jee/PAPERS/display.cfm?pdf=922.pdf&special_issue=922

[9] Dey, E. L., Herrin, G. D., Veenstra, C. P., "A Model for Freshman Engineering Retention", *Advances in Engineering Education*, American Society for Engineering Education, Winter, 2009, http://advances.asee.org/vol01/issue03/papers/aee-vol01-issue03-p07.pdf.

[10] "Engineering Student Retention: Reducing Attrition and Improving Graduation Rates of Upperclassmen Engineering Students, An Annotated Bibliography", *Polson Enterprises*, August 2003, http://www.virtualpet.com/engineer/retent/retent.htm.

[11] ALEKS® Corporation: About us, http://www.aleks.com/about_us.

[12] "Calculus Readiness Requirement", *The University of Texas at Austin Cockrell School of Engineering,* http://www.engr.utexas.edu/undergraduate/admission/489-calculus-readiness-requirement.

[13] "Connecting College Readiness Standards™ to the Classroom for Mathematics Teachers", *ACT*, 2008, http://act.org/standard/guides/act/pdf/ACT_Math.pdf.

AUTHOR INFORMATION

Ed Wheeler Professor and Interim Chair, Department of Engineering, University of Tennessee at Martin, Martin, TN 38238, ewheeler@utm.edu.