An Elective Course in Rocketry

Paul F. Penko
Physics Department
Baldwin Wallace University

Abstract

A course in rocketry is offered through the Physics Department at Baldwin Wallace University (BWU) as a 1-hour elective in an 8-week period, termed a minimester. The objective is for students, working in teams of 2 or 3, to design, analyze, build and launch rockets. The rockets must be designed to meet the maximum-allowable altitude for the launch site as specified by the FAA. Launches are done under the auspices of the National Association of Rocketry (NAR).

Course Description

The course meets once a week for 1 hour 40 minutes for 8 weeks. Students are required to have had the first series of physics and calculus, though not necessarily differential equations. Material covered includes trajectory analysis by solution of the differential equation of Newton's Second Law of motion, accounting for change of mass and drag variation with velocity. The equation is solved by means of finite differencing. The principle of stability based on center-of-mass and center-of-pressure is covered, as well as some basic rocket design.

Students are required to perform a trajectory and stability analysis, and produce a design, specified in an engineering drawing. Each team is allotted a budget of $300. The design is required to include an altimeter as payload and can include other instruments at the discretion of individual teams. Rockets are designed to attain maximum allowable launch-site altitude and maximum allowable motor size. Status presentations are made at mid-term and final designs presented at end-of-term along with a final written report. A parts list is included for purchase by the instructor. The course is graded on a satisfactory/unsatisfactory basis. Actual rocket construction takes place after completion of the course on students' own time. University shop equipment is available for their use under supervision. Because of the early onset of winter weather this school year, launches will be done in spring term.

Summary

This course was offered three times over a period of several years, twice at The Ohio State University (OSU), Aerospace Engineering Department, and most recently at BWU through the Physics Department. The students at OSU were juniors and seniors, mostly aerospace or mechanical engineering majors, so had all necessary requisites for the course. The students at BWU were junior and senior physics majors, and too, had the necessary math and physics requisites. The course offering at OSU was given over a quarter, i.e. 10 weeks, and allowed more time for construction than does the 8-week minimester at BWU. Conducting the course in 8 weeks does require students to do the construction after the term has ended. The course at OSU was offered in winter quarter such that the launch directly followed in spring quarter. Doing the course in minimester A, i.e. at the beginning of fall semester, does necessitate a delay of nearly 4-5 months until launch, given northern Ohio weather.

Generally speaking, the course was favorably received by students. Their overall assessment is that it gives them a "real world" experience, an actual engineering application of physics and analysis. One criticism from this past term is that insufficient time was spent on actual rocket-design and construction details. This deficiency will be addressed in future offerings of the course.

The paper is organized such that anyone interested in pursuing the technical area as a course offering has the analysis and references available. Any use of the material herein does not require the author's permission.

Trajectory Analysis

Newton's Second Law applied to a rocket is,

$$\sum F = \frac{d}{dt}(MV)$$

or

$$T - D - Mg\cos\theta = \frac{d}{dt}(MV) \qquad (1)$$

Here, T = thrust, D = drag, M = mass, V = velocity, θ = flight angle, g = acceleration of gravity and t = time.

This equation can be solved, without simplifying assumptions, by finite-difference time-stepping. Here, M, V, D, and T, are considered to vary in time. Variation of thrust is obtained from motor test data provided by the manufacturer.

Basic Time-Stepping

A basic time-stepping method is employed to solve differential equation (1) by first writing it in finite-difference form,

$$\frac{(MV)_{i+1} - (MV)_i}{t_{i+1} - t_i} = T_i - D_i - M_i g\cos\theta_i$$

where, $i = 0, 1, 2, \ldots, n$

with the initial conditions, $t_0 = 0,\ V_0 = 0$

$\theta_0 =$ initial launch angle,

$M_0 = M_I =$ initial rocket mass,

$T_0 =$ initial motor thrust.

Solving for (MV) at the new time step, $i+1$,

$$(MV)_{i+1} = (MV)_i + [T_i - D_i - M_i g\cos\theta_i]\frac{\Delta t_i}{M_i}$$

and for the velocity,

$$\boxed{V_{i+1} = \left\{(MV)_i + [T_i - D_i - M_i g\cos\theta_i]\Delta t_i\right\}/M_{i+1}} \qquad (2)$$

where $\Delta t_i = t_{i+1} - t_i =$ time increment,

$M_{i+1} = M_i - \dot{m}_p \Delta t_i$,

$\dot{m}_p = M_P/t_B \rightarrow$ assumed constant (usually not sufficient info to do otherwise).

The drag force can be given by,

$$D_i = \tfrac{1}{2} C_D \rho A V_i^2 \qquad (3)$$

where $\rho =$ air density (assumed about constant over the trajectory),

$C_D =$ drag coefficient, assumed constant,

$A =$ frontal area.

The thrust, T_i, is obtained from a thrust vs. time curve as in Appendix A. Drag coefficient as a function of angle-of-attack, α, and velocity is given in Appendix B.

The altitude gained in each time increment Δt_i is,

$$\Delta h_{i+1} = \tfrac{1}{2}(V_{i+1} + V_i)\Delta t_i \qquad (4)$$

where the average velocity is used over the time increment.

For the glide portion, i.e. unpowered portion, of the flight, eq. (2) reduces to,

$$\boxed{V_{i+1} = V_i - \left[D_i + M_i g\cos\theta_i\right]\frac{\Delta t_i}{M_i}} \qquad (5)$$

The total gain in altitude is, $$\boxed{H = \sum_{i=1}^{n} \Delta h_i} \qquad (6)$$

An example of results from a student team on their trajectory analysis is given in the following table, where H_B = altitude at end of motor burn, H_A = altitude gained during free-flight and H_{TOT} = final altitude. In the analysis, a constant coefficient-of-drag of 0.5 was assumed. A single AeroTech G-80T motor is used with an average thrust = 80.35 N.

Table 1. Trajectory Analysis

Mass of Rocket	0.354	kg	ρ(air density)	1.2	kg/m³	Coefficient of Drag C_D	0.5	
Mass of Propellant	0.0625	kg	dm/dt	0.03676	kg/s	Trust T (constant)	80.35	N
$A_{gravity}$	9.8	m/s²	t(burn)	1.7	s	Mass ratio	0.85	
θ	0		Frontal area A	0.0034211	m²	Total Impulse	136.6	N-s

Time step analysis

Time step analysis
$V_{i+1}= V_i+[T_i-D_i-M_i g\cos\theta_i](\Delta t/M_i)$
$D_i=1/2*C_D\rho AV_i\text{^}2$
$M_i+1= M_i-dm/dt*\Delta t_i$
$dm/dt=M_P/t_B$

H_B	H_A	H_{tot}	
230.71	483.50	714.21	m
V_{burn}	231.4032	m/s	

A drawing of the rocket for this case is given in Appendix C.

Stability Analysis

Induced lift and drag on the rocket body, where lift is conventionally normal to the direction of motion, i.e. the velocity vector, $\vec{V}$, and drag is coincident with the direction of motion, is illustrated in Fig. 1. Lift, L, and drag, D, are related to the velocity, V, by,

$$D = \tfrac{1}{2} C_D \rho A V^2$$

$$L = \tfrac{1}{2} C_L \rho A V^2$$

where C_D = the drag coefficient as before,
C_L = the lift coefficient.
ρ = air density.

The resultant lift force, $\vec{L}_{RN}$, from the lift that occurs when the rocket axis is at an angle-of-attack, α, acts through the center-of-pressure, x_{cp}, and normal to the rocket axis, where,

$$\vec{L}_{RN} = \vec{L}_{PN} + \vec{L}_{FN}$$

Here $\vec{L}_{PN}$ = lift from pressure distribution over rocket body normal to rocket axis,
$\vec{L}_{FN}$ = lift from fins normal to rocket axis,
$\vec{L}_{RN}$ = resultant lift acting through the center-of-pressure, x_{cp}, normal to rocket axis.

The location of x_{cp} is such that $\vec{L}_{RN}$ produces the same moment about the center-of-mass, x_{cm}, as the forces $\vec{L}_{PN}$ and $\vec{L}_{FN}$, i.e.,

$$\vec{L}_{RN} \times \vec{l}_R = \vec{L}_{PN} \times \vec{l}_P + \vec{L}_{FN} \times \vec{l}_F \qquad \text{(vector cross product)} \qquad (7)$$

where $\vec{l}_P$ = displacement from the body-lift vector to *cm*
$\vec{l}_F$ = displacement from the fin-lift vector to *cm*
$\vec{l}_R$ = displacement from the resultant lift vector to *cm*.

With respect to the center-of-mass, x_{cm}, the resultant force acting through the center-of-pressure, x_{cp}, will give a moment,

$$\vec{M} = \vec{L}_{RN} \times \vec{l}_R$$

where $l_R = x_{cp} - x_{cm}$ (scalar magnitude)

If x_{cp} and x_{cm} are <u>not</u> coincident, a pitching moment acts on the vehicle and the main stabilizing force is $\vec{L}_{FN}$, the force of fin lift. For stability, the relation between the lift from pressure distribution on the body of the rocket, $\vec{L}_{PN}$, and lift from the fins, $\vec{L}_{FN}$, i.e. where the fin-lift moment counteracts the body-lift moment is,

$$\left|\vec{L}_{FN} \times \vec{l}_F\right| \geq \left|\vec{L}_{PN} \times \vec{l}_P\right| \qquad (8)$$

Dropping vector notation and using magnitudes of force and distance, and the right-hand-rule for moment sign, i.e. scalar notation,

$$-L_{RN}\left(x_{cp} - x_{cm}\right) = L_{PN} l_P - L_{FN} l_F$$

or

$$\left(x_{cp} - x_{cm}\right) = -\frac{L_{PN} l_P - L_{FN} l_F}{L_{RN}} = \frac{L_{FN} l_F - L_{PN} l_P}{L_{RN}}$$

But from (8),

$$\left(x_{cp} - x_{cm}\right) \geq 0$$

therefore,

$$\boxed{x_{cp} \geq x_{cm}} \qquad (9)$$

This states that the location of center-of-pressure, x_{cp}, should be <u>aft</u> (behind) or below, the center-of-mass, x_{cm}, or (ideally) <u>coincident</u> with the center-of-mass for stable flight (see Fig. 2). In this case, the lift produced by the fins will always overcome the moment from body lift.

The Barrowman equations (Ref. 4) are employed to determine center-of-pressure, x_{cp}. Center-of-mass can be determined by a simple balance test or by,

$$x_{cm} = \sum_1^n x_{cmn} m_n \Big/ \sum_1^n m_n \qquad (10)$$

where x_{cmn} = center-of-mass for component *n*;
m_n = mass of component n.

References

1. Penko, P.F., Rocketry Class Notes, PHY-263, Fall Semester 2013.
2. Stine, G.H., Stine, W., Handbook of Model Rocketry, John Wiley & Sons, 2004.
3. Sutton, S.P, Biblarz, O., Rocket Propulsion Elements, John Wiley & Sons, 2010.
4. Barrowman, J.S., National Aeronautics and Space Administration, The Practical Calculation of the Aerodynamic Characteristics of Slender Finned Vehicles, March, 1967.

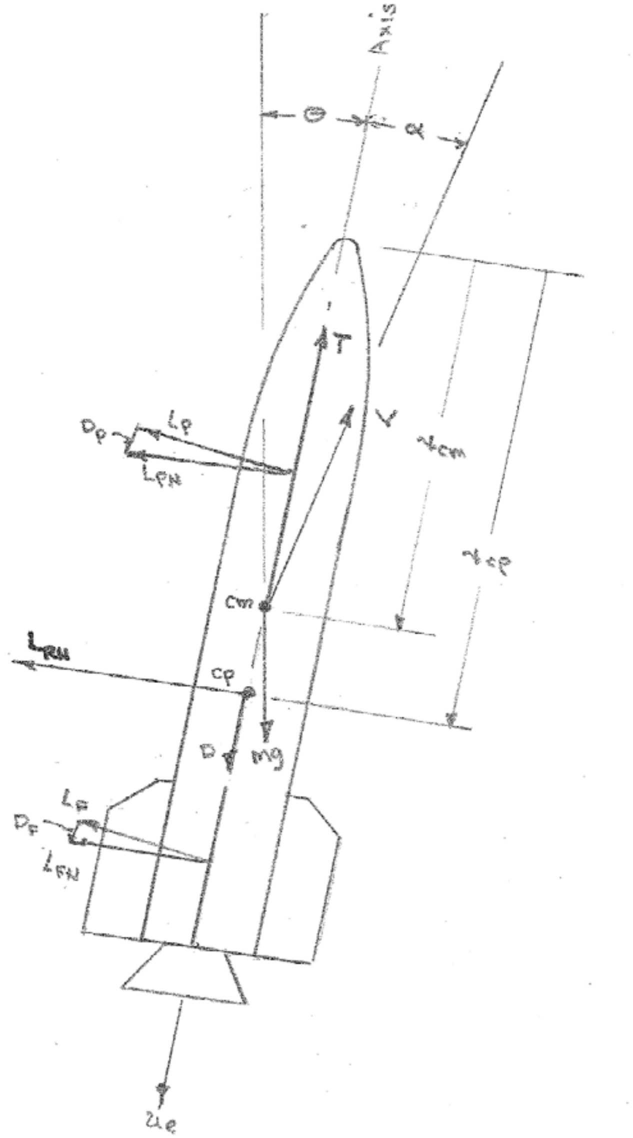
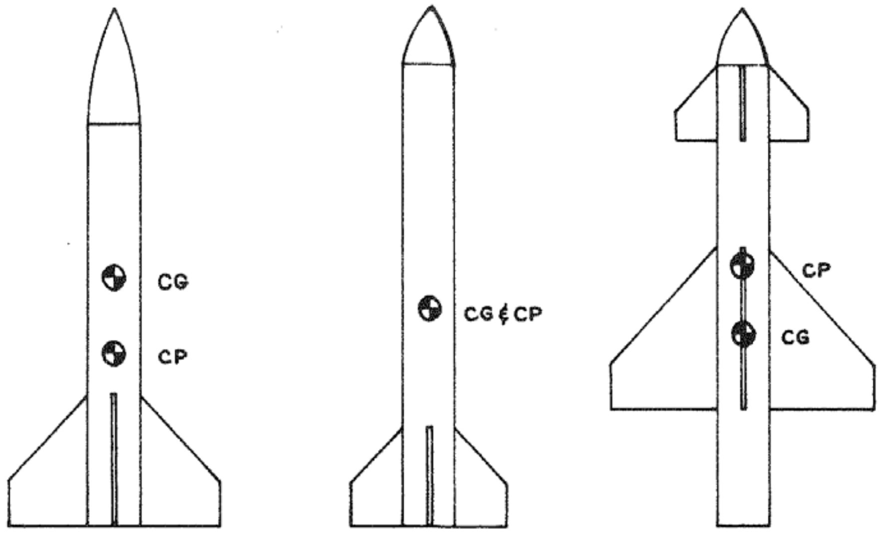


Fig. 1. Vector diagram for stability analysis.



Unstable Neutral Stable

Note: *CG & CM* are synonymous.

Fig. 2. The three conditions of stability. From Handbook of Model Rocketry

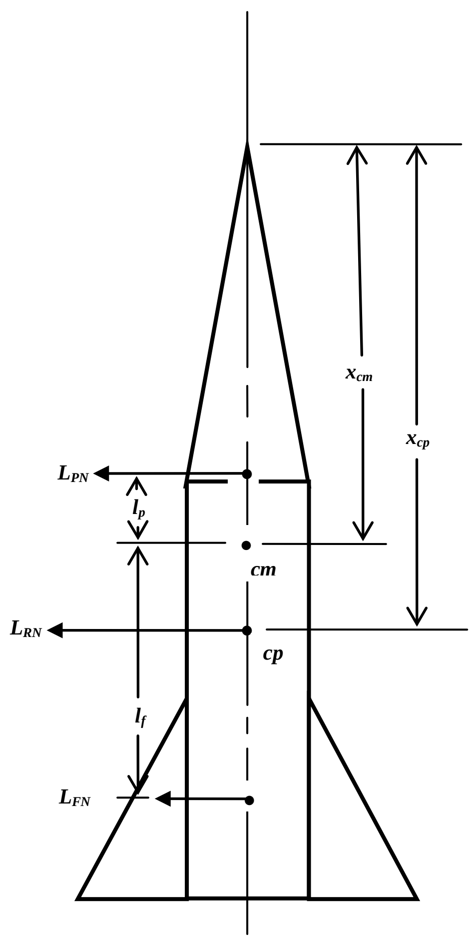
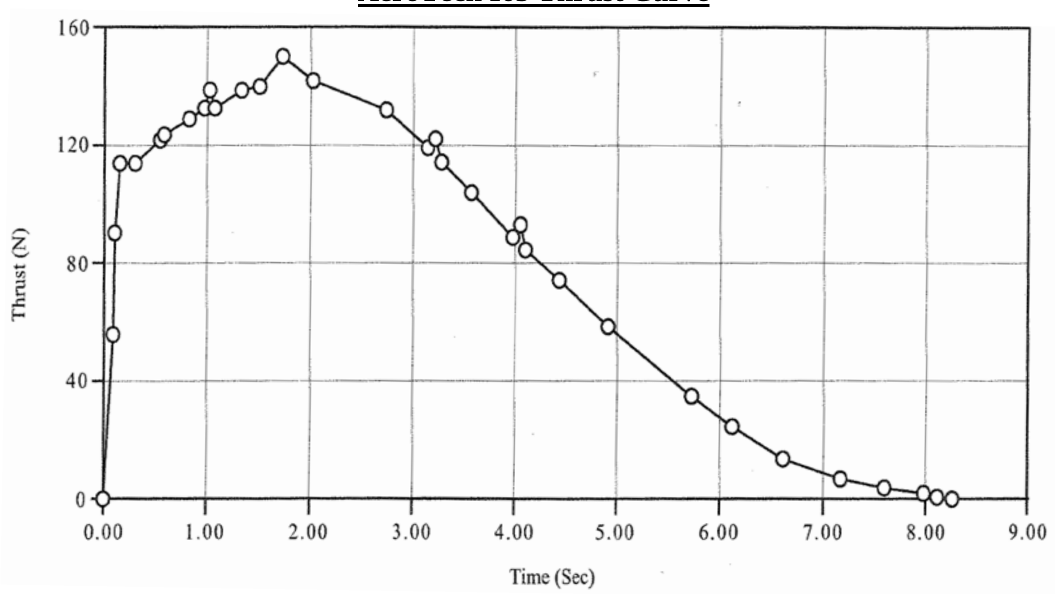


Fig. 3. Definition of dimensions for stability analysis.

Appendix A

AeroTech I65 Thrust Curve



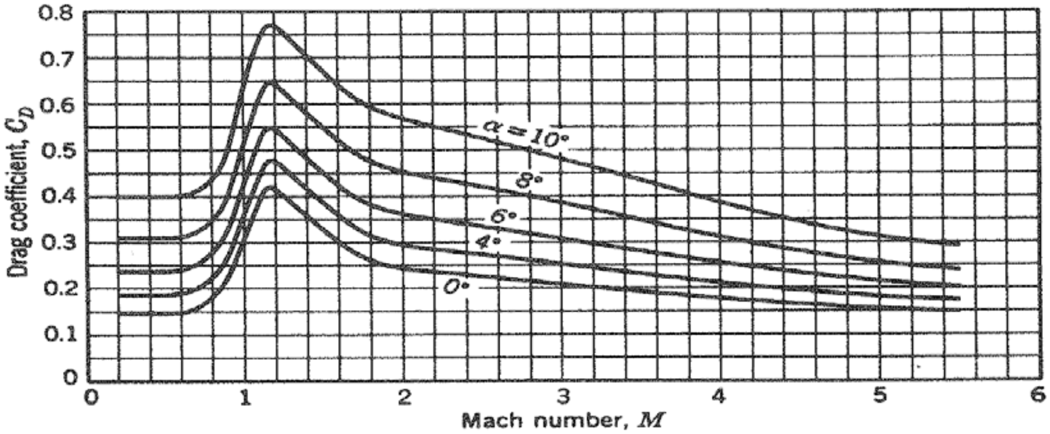
Thrust Data

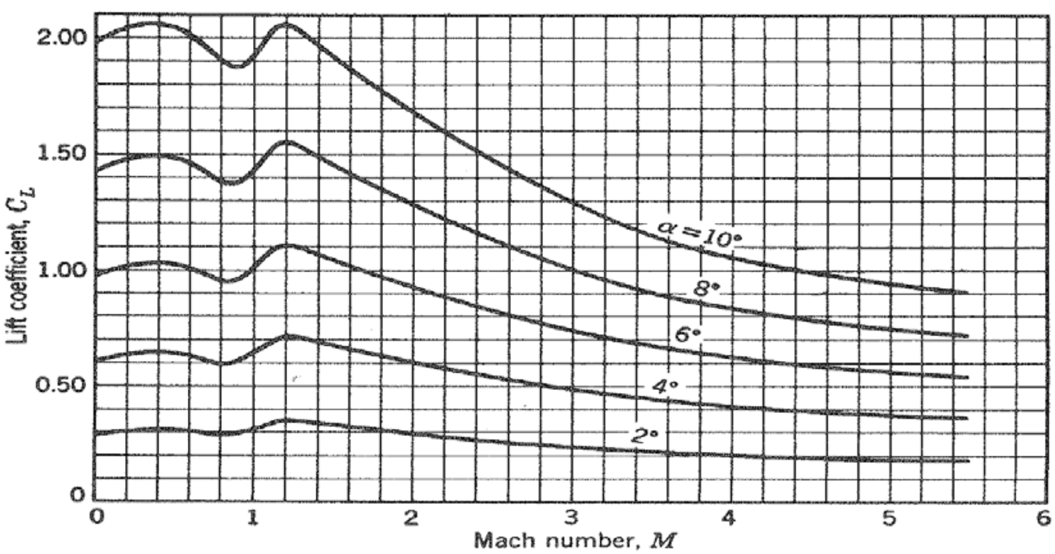
Time	Thrust
0.086	55.739
0.099	90.267
0.148	113.911
0.296	113.911
0.542	121.767
0.579	123.588
0.825	128.953
0.973	132.593
1.022	138.725
1.071	132.593
1.330	138.629
1.502	139.875
1.724	150.070
2.019	141.886
2.733	131.923
3.140	119.181
3.213	122.151
3.275	114.295
3.570	103.948
3.977	88.791
4.051	93.036
4.100	84.547
4.432	74.229
4.912	58.440
5.725	34.758
6.119	24.430
6.611	13.489
7.166	6.787
7.596	3.729
7.978	1.886
8.114	0.665
8.260	0.000

Appendix B

Lift & Drag Coefficients

- Following are lift and drag coefficients for the V-2 rocket as a function of Mach Number*, M, and angle-of-attack, α,





From Rocket Propulsion Elements, Sutton & Biblarz, 8th Edition, 2010.

* Note: $M = V/S$

V = vehicle velocity, m/s

$S = \sqrt{\gamma RT}$ speed of sound, m/s

$\gamma = 1.4,$ ratio of specific heats, dimensionless

R = 8310 J/kg_{mol}-K

T = temperature, K

Appendix C

Rocket Dimensions