# Work in Progress - Analyzing the Flowrate of Water Through a Community Sand Filter Using Engineering Numerical Software

Sara Rimer, Kumar Yelamarthi
Central Michigan University, rimer1sp@cmich.edu, kumar.yelamarthi@cmich.edu

***Abstract* – Due to its efficiency when removing pathogens and particles, slow sand filtration has been an attractive means of purifying water for many years. Also, it is ideal for developing communities due to its absence of requirement for any machinery, and higher material life. Thus, slow sand filtration is an effective means of treating water in many households and smaller communities worldwide. When designing and implementing a slow sand filter, it is important to determine the amount of water a filter can potentially supply at any given time, which is the focus of this research. Because of the water's flowrate dependency on the change of its vertical position and time, the solution to the flowrate of water in a sand filter can be expressed as a partial differential equation (PDE). To numerically solve the problem, the PDE toolbox feature of MATLAB and the optimization tool of Microsoft Excel Solver has been used.**

*Index Terms* – Fluid dynamics, Optimization, Partial differential equations, Sand filtration.

## INTRODUCTION

The purpose of this project was to develop a partial differential equation (PDE) in the field of environmental fluid dynamics and evaluate it with MATLAB, Microsoft Excel, and other computing software technologies. The PDE developed was based on knowledge gained from fluid dynamics and environmental engineering. More specifically, it dealt with water purification with the use of sand filtration.

Through slow sand filtration, water is filtered in two ways: (i) by a biological layer and (ii) by a porous medium. This research is primarily concerned with the filtration of the water through the porous medium. Using Darcy's Law of fluid flow through a porous medium, for a given hydraulic conductivity of the sand medium and a size of a sand filter, the gradient of the flow rate of the water in the vertical direction as it passes through the porous medium can be calculated. The flowrate of the water is then used to calculate the amount of water provided for a given amount of time. Since slow sand filtration uses various types of materials to comprise its medium (e.g. sand, gravel), the hydraulic conductivity of the medium is a composite of the different hydraulic conductivity constants of each layer of material. With time, as water is continually filtered through the medium, it accumulates within the medium and a biofilm grows between sand grains, changing its hydraulic conductivity with respect to time. The change in the hydraulic conductivity of the medium with respect to time and vertical position of the water, also affects the flowrate of the water, in turn making the flowrate dependent on time and vertical position.

The results presented in this paper are the beginning of a larger research project. The results are based on a differential equation that has been developed, with plans concerning the development of the PDE discussed at the end of the paper. Other possibilities for the future work of the project are discussed at the end as well.

## BACKGROUND

### *I. Sand Filtration*

Sand filtration can be one of the cheapest and effective forms of treating drinking water [1]. It is a technology used worldwide, and often times, is one of the most transferrable technologies to developing communities. For this reason, it is important to study sand filtration and the limitations it may pose for communities. One such limitation that could be posed is the amount of water sand filtration can supply to a community for a given time. To do this, the flowrate of water in a sand filter can be examined with the use of Darcy's Law.

### *II. Darcy's Law*

In general, the study of water flow through a sand filter uses the same methods as water flow through ground water. Thus, the flow of water through the control volume uses Darcy's Law [2] as in (1):

$$Q = AK\frac{\Delta h}{L} \tag{1}$$

where $Q$ is the volumetric flowrate of the water, $A$ is the cross-sectional area of the control volume, $K$ is the hydraulic conductivity of the sand, $\Delta h$ is the change in head, and $L$ is the length of the sand filter. For this project, the control volume of the sand filter includes all of the sand within the filter excluding the biological layer that forms at the top of the filter.

## DEVELOPMENT OF EQUATION

Usually, Darcy's Law can be treated as a PDE if the head changes in more than one direction. However, because the fluid of a sand filter only travels in one direction due to gravity, Darcy's law is only a one-dimensional differential equation. Thus, in order to make Darcy's Law a PDE, a different parameter of the equation would need to be evaluated differentially in more than one dimension. Thus, hydraulic conductivity was examined.

Hydraulic conductivity, which is the ability of a specific fluid to flow through the pores of a material, can change with time [3]. This is because as time passes, and more and more water is filtered through a sand filter, a biological film begins to grow around the grains of the sand. Thus, as the grains of the sand grow, the ability of water to flow through the medium changes, and so does the hydraulic conductivity. Usually, hydraulic conductivity of various types of sand is found empirically and many times when working with sand filters, the hydraulic conductivity is taken to be constant. However, for this project, it was desired to have the hydraulic conductivity of the sand change with time. Thus, the Kozeny-Carmen Bear [3] as in (2) was employed:

$$K = \left(\frac{\rho_w g}{\mu}\right)\frac{n^3}{(1-n)^2}\left(\frac{D_m^2}{180}\right) \tag{2}$$

where $\rho_w$ is the fluid density, $g$ is gravity, $\mu$ is the fluid viscosity, $D_m$ is the grain size, and $n$ is porosity. Thus, (1) becomes:

$$Q = A\left(\frac{\rho_w g}{\mu}\right)\frac{n^3}{(1-n)^2}(D_m{}^2)\left(\frac{\Delta h}{180L}\right) \tag{3}$$

and (3) becomes the following when the diameter of the grains of sand grow with respect to time:

$$\frac{dQ}{dt} = A\left(\frac{\rho_w g}{\mu}\right)\frac{n^3}{(1-n)^2}\left(\frac{dD_m}{dt}\right)^2\left(\frac{\Delta h}{180L}\right) \tag{4}$$

Because the grain size is the transient parameter, an arbitrary growth equation with respect to time could be used to represent the change in grain size. Also, as grain growth is believed to become constant with respect to time, a negative inverse equation was used to represent this growth. The specifics of the equation chosen are discussed in the next section.

In order to make (3) into a PDE, the hydraulic conductivity would need to be differential with respect to another dimension as well. For this equation, the plan was to have to the conductivity differential in a space dimension in which the conductivity would be differential with respect to height of the sand filter. For simplicity, the sand medium in the filter is assumed to be the same. However, in general, usually sand filters have many different layers of earth of varying conductivity that assist in the filtration of water as in Figure 1.

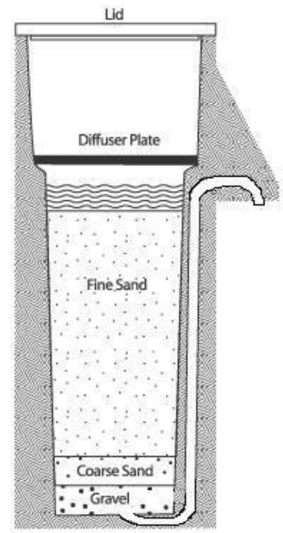


FIGURE 1: HOUSEHOLD SAND FILTER SCHEMATIC WITH MULTIPLE LAYERS OF EARTH [4 ]

Thus, the change in conductivity with respect to the height of the filter can easily be represented by a stepwise function or approximated using a continuous and differentiable function (e.g. a polynomial). Developing and solving this differential equation is the goal of this project and will be carried out in the future.

## RESULTS

The idea for this project came about through the student's work with the Engineers Without Borders-Central Michigan University (EWB-CMU) chapter. The population that these sand filters would serve includes developing communities and/or households within those communities. The EWB-CMU chapter on their assessment trip to Sainte Luce, Madagascar saw household sand filters utilized within the community. A picture of a household sand filter can be seen in Figure 2.

FIGURE 2: TYPICAL HOUSEHOLD SAND FILTER IN SAINTE LUCE, MADAGASCAR

In order to utilize MATLAB and find solutions to the given equation, a case study for a specific sand filter has been conducted. Accordingly, a typical household sand filter used in Sainte Luce is used in this paper. This study can easily be applied to any slow sand filter of any size.

The volume of sand within the filter has dimensions of 250 mm for the diameter and 650 mm for the height, *L*. Thus, with a diameter of 250 mm, the cross-sectional area, *A*, is 196350 $m^2$. The sand used for the filter was medium sand with a grain size $D_m$ of about 0.25 mm. The fluid viscosity $\mu$ of the dirty water was assumed to be that of water at 21° C, which is $1.002\times10^{-3}\ N\cdot s/m^2$. Gravity *g* is assumed to be $9.81 m/s^2$ and the fluid density $\rho_w$ at 20° C is $998.2\,kg/m^3$. The porosity of the sand *n* (i.e. the percentage of the volume that the sand does not fill) is assumed to be 0.3 [3].

For the change in head with respect to distance, it can be assumed that for a volume of sand above ground, the head is only the height of the water above the ground. Thus, the change in head is merely the change in height of the water as it moves through the sand filter. To represent this change, the basic linear equation in (5) is used.

$$\Delta h = h \tag{5}$$

Here, *h* is the distance of the water above the bottom of the filter. This equation is only valid for distances between 0 mm - 650 mm (i.e. within the height of the sand filter).

As stated earlier, an arbitrary growth equation could be used to describe the change of the sand grain size with respect to time. As the growth of the biofilm within the control volume is believed to level out with time, the growth equation that was chosen to represent the growth is a negative inverse equation. Using (3), the hydraulic conductivity of medium sized sand at 0.25 *mm* should be about $1.9\times10^{-4}\ m/s$. According to [2], for medium sized sand, the hydraulic conductivity is about $1.0\times10^{-3}\ m/s$. Using (3) with *K* equal to $1.0\times10^{-3}\ m/s$ and solving for the diameter, $D_m$, gives $D_m = 0.35$ *mm*. Thus, for this growth equation, as time approaches infinity, *K* should equal $1.0\times10^{-3}\ m/s$ with the diameter of the sand grain approaching 0.35 *mm*. Thus, the growth equation chosen for this was such that at the initial time, the sand diameter size is 0.25 *mm* and as time approaches infinity, the grain size is at 0.35 *mm*, which is the following (in meters):

$$\frac{dD_m}{dt} = -\frac{1}{t+0.0001} + 0.035 \tag{6}$$

## I. MATLAB

Using MATLAB and graphing (4) with the use of Equations (5) and (6), yields the following result:

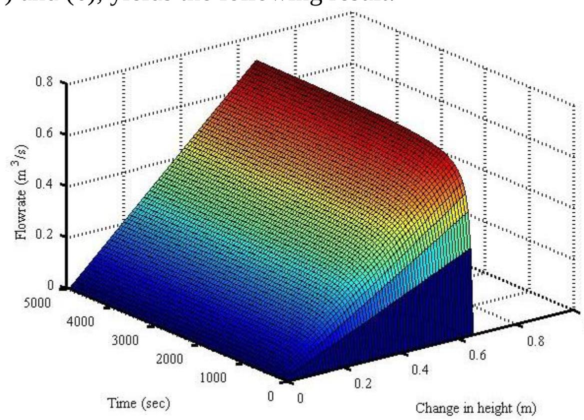


FIGURE 3: CHANGE IN FLOWRATE OF WATER WITH RESPECT TO TIME AND LOCATION

Using (6), the initial grain size of 0.25 *mm* occurs at about 29 *sec* while the maximum grain size of 0.35 *mm* is reached at about 4000 *sec*. As stated earlier, the growth equation is arbitrary and was used primarily to exemplify how (2) affects (1). Further analysis is needed in order to develop a better equation that accurately represents the growth of the biofilm around the sand grains.

## II. Microsoft Excel Solver

Microsoft Excel Solver was used to carry out optimization analysis of the equation. The optimization analysis was carried out to find the values of head and of time that would lead to an optimized flowrate. In order to have the solver converge to a solution, constraints were given to the head and time values. The time constraint was between 0 and 11000 *sec* and the head constraint was between 0 mm - 650 mm. Because after a certain amount of time, the change in growth of the sand grain will disappear, a large value for the upper bound for the time constraint was arbitrarily chosen so that the growth will have already disappeared. The results are presented in Table I.

TABLE 1: RESULTS FROM MICROSOFT EXCEL SOLVER

| Variable | Unit | Value |
|---|---|---|
| Density | $kg/m^3$ | 998.2 |
| Viscosity | $N*s/m^2$ | 1.002E-03 |
| Gravity | $m/s^2$ | 9.81 |
| Porosity | -- | 0.3 |
| Length | m | 0.65 |
| Cross-sectional area | $m^2$ | 0.1963 |
| Diameter of sand grain | m | 0.0349 |
| Head | m | 0.65 |
| Time | sec | 11000 |
| ***Flowrate*** | ***$m^3/s$*** | ***0.715667686*** |

As expected, because the higher the head value (i.e. the greater the change of the water's position with respect to the top of the filter) the greater the flowrate, 0.65 *m* was the value of head that maximized the flowrate. Also, because the flowrate becomes constant after the change in diameter of the sand grain disappears and because 11000 *sec* optimized the flowrate, it shows that the largest size the sand grain will become is what will maximize the flowrate.

The Microsoft Excel Solver optimization tool would be more useful for this project if it was helping with the design of the sand filter. When designing a community sand filter, it might be more useful to solve (1) for the sand filter dimensions (e.g. cross-sectional area, length of sand filter) rather than for the flowrate of the water. Thus, for example, the MS Excel Solver can optimize the flowrate based on different dimensions of the sand filter.

## FUTURE WORK

As stated earlier, the ultimate goal of the project is to develop and numerically evaluate a partial differential equation through the use of various engineering software. Thus, making the flowrate equation differential with respect to another dimension is the next step of the project. This will be done by either making the hydraulic conductivity differential with respect to height (as was discussed earlier) or taking the head to be differential with respect to time [5]. Later, the equation would be numerically solved with the use of Matlab's Partial Differential Equation toolbox.

Also, for future work of this project, it is desired to make the equation able to assist in the design of the sand filter. Currently, the Darcy's Law is solved for its flowrate and is dependent upon a fixed sand filter size. It would be useful to solve the equation for the cross-sectional area and height of the sand filter based on a given flowrate that is determined by the demand for water of a community. Thus this project could help assist in the design aspect of a sand filter.

## REFERENCES

[1] Davis, J. and Lambert, R. Engineering in Emergencies. ITDG, 2002.

[2] Bear, J. Dynamics of Fluids in Porous Media, Dover, 1988, 136.

[3] Schwartz, F.W. and Zhang, H. Fundamentals of Ground Water. Wiley, 2008, 53.

[4] Photograph. Water for Resettled Program. GlobalMed: David McAntony Gibson Foundation. Web. 12 Dec. 2009. <http://www.globalmedic.ca/missions/SriLanka/aid_march_2009/biosand.html>

[5] Parkhurst, D. F. Introduction to Applied Mathematics for Environmental Science, Springer, 2005, 250-283.

## AUTHORS

**Sara Rimer,** Mechanical Engineering and Mathematics student, Central Michigan University, College of Science and Technology, Mount Pleasant, Michigan, 48859, rimer1sp@cmich.edu. She is also a member of the Engineers Without Borders - Central Michigan University student chapter.

**Kumar Yelamarthi** is an Assistant Professor in Electrical Engineering. His current research interests are in the areas of RFID, VLSI, Computer Aided Design tool development, digital design automation, autonomous adaptive systems, applied electronics, and engineering education. He has served as a technical reviewer for several IEEE/ASME/ASEE international conferences and journals, and has published over 50 articles. He is an elected member of Tau Beta Pi and Omicron Delta Kappa honor societies.