Successful Collaboration among Engineering, Education, and Mathematics

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The purpose of this report is to share the ongoing research activities of a group of mathematics educators whose sole focus is improving mathematics teaching and learning for engineering students. Our vision is to provide experiences for engineering students that showcase the relationships between mathematics, the physical sciences, and engineering problems. We do this through using mathematical modeling approaches to demonstrate how mathematics and engineering techniques can be combined to solve problems, using expert cognitive modeling to demonstrate mathematical modeling skills, and adopting an anywhere/anytime attitude toward student learning (asynchronous via Adobe Connect and Livescribe's Pencasts). To help us realize our goals of increasing accessibility of our mathematics content and promoting multidisciplinary learning, we received a grant from The Ohio State University's (OSU) College of Engineering. In this paper, we describe the program of research we developed in coordination with the College of Engineering. The value of our contribution is a demonstration of the kinds of work and benefits possible when multidisciplinary collaboration occurs in education. We hope that other universities might adopt a similar multidisciplinary vision for collaboration among departments responsible for the education of engineering students.

Context

Our research is centered on the redesign of a differential equations course for science and engineering majors. The course is critical for many engineering disciplines because it provides the foundations for studying systems that change over time. There is anecdotal and research-based evidence that the kinds of knowledge gained from traditional skills-driven courses are not valued as highly as skills like modeling [1] or conceptual knowledge [2]. Indeed, faculty recommend that service courses in mathematics be made more relevant to their students by incorporating an engineering viewpoint [3, 4]. Taught in a skills-oriented manner, students come away with the idea that differential equations are no more than a series of methods [5,6] rather than an overview of the subject [7] and its utility. Members of the mathematics, education, and engineering faculty at The Ohio State University came together to address this problem.

In order to respond to the engineering faculty's needs for this course, mathematics faculty began to organize the differential equations content around pragmatic problems in science and engineering. Students in the differential equations course for science and engineering majors are typically in their second year. They have completed calculus and a first year of general engineering courses. A set of differential equations problems drawing on first-year engineering and science contexts were developed in order to highlight key mathematical insights [8]. To do this, we used a modeling approach (described below). The text proceeds through each chapter by presenting an in-depth discussion of a lifelike example (usually a derivation), a discussion of the abstract view of the example, and then a series of progressively more challenging (and less structured) exercises.

The text content was created with the expertise of mathematics faculty with attention to mathematics education research on students' thinking about the concepts supporting differential equations. To accomplish this, mathematics educators collaborated with the mathematics faculty through classroom observations and through weekly meetings where we discussed classroom events, theories of learning and knowledge from mathematics education research, and factors impacting engineering students' success from engineering education research. We also attended bi-weekly engineering education seminars held by the Engineering Education Innovation Center (on OSU's campus) in order to share our students' successes and difficulties with the engineering and engineering education faculty. This gave us a broader view of the needs of the students in terms of their overall collegiate education.

In addition, because of our work with the students, we were awarded a grant from the College of Engineering to introduce and develop our ideas for technology use in the classroom. We used the grant to purchase instructional software and hardware (Livescribe Smartpen, peripheries, and a tablet computer) and to conduct educational research into students' adoption of the media and their thinking about the subject matter.

Mathematical Modeling Approach

A *model* is a simplified representation of some system. *Modeling* is both a sequence of behaviors and a way of thinking about a problem [9]. In *mathematical modeling*, one must find a way to describe, explain, or interpret phenomena in mathematical terms [10]. In the educational research literature, mathematical modeling is theorized as a cyclical, iterative process that connects the Real World to the Mathematical World [11]. A problem in the real world must be conceptualized as a well-posed mathematical problem, analyzed mathematically, and the solution must be interpreted in terms of real world constraints. Mathematical modeling is difficult and messy: it requires identification and justification of the inclusion of variables, assumptions, and constraints. Complicating the issue is the fact that typical mathematics classrooms focus only on the mathematical analysis.

In addition, prior educational research in differential equations suggested that students enrolled in a conceptually-oriented differential equations class performed similarly to students in a traditional skills-oriented class on procedural tasks, but performed better on conceptual tasks [12]. Therefore, we sought to design instruction and supporting materials around two principles: (i) identifying and reinforcing conceptual strands throughout the content, and (ii) emphasizing mathematical modeling as practice for solving lifelike problems.

In the classroom and in the text, students were shown and also asked to solve paradigmatic problems in differential equations that arose from engineering contexts. For example, instead of teaching them to identify and solve first-order linear equations, the students were asked to consider the following system: a dam and its reservoir being fed by a contaminated stream. The students were asked to model the contaminant in the reservoir. Instead of presenting myriad analytic techniques, the students were shown an educated guess-and-check solution technique similar to the method of undetermined coefficients. Using principles like linearity and superposition, the students constructed homogeneous solutions from exponential functions and added in particular solutions found by considering the role of a forcing term present the physical system. These didactic choices kept the focus on the analysis *possible* with mathematics, rather than on largely algebraic analytic techniques. A more detailed explanation of the differences between the texts is offered elsewhere [13].

A small-*n* study was conducted to get a sense of the efficacy of the approach [14]. Two classrooms – one using the traditional skills-based approach ($n = 30$) and one using the conceptually-based approach ($n = 21$) – were given matched items on their final exams. When taking the statistically significant previous mathematics achievement into account ($F = 25.445$, $df = 1, 47$, $p < .001$), the students in the classroom with the conceptual, modeling approach performed better ($F = 5.972$, $df = 1, 47$, $p = .018$, $\eta^2 = .113$, observed power = .668). Especially interesting is that the prior mathematics achievement of the students in the conceptually-oriented class was collectively lower. A number of factors could contribute to this result, such as the inclusion of engineering viewpoints and mathematical modeling, the focus on only one conceptual solution technique, or the problem-based instructional approach. In any case, the results were encouraging and we sought to provide more support for the students to learn about expert mathematical modeling.

Cognitive Modeling Approach

The redesigned differential equations curriculum is challenging for students. One aspect that they find difficult is the focus on concepts and modeling, as opposed to algebraic manipulations. We sought a way to both demonstrate to them the habits of mind that we envisioned and to provide further instructional support. To this end, we selected Livescribe's Smartpen technology to aid us in developing cognitive models of the professor's “expert” modeling activity. Challenging modeling problems from the new text's exercises were selected to be transformed into narrated worked examples.

Mathematical examples are commonplace in textbooks, either as worked examples or as exercises, and so are an integral part of how students interact with the material. A worked example consists of a problem statement, a sequence of steps leading to a solution, and the final solution. Research has suggested that textbooks encourage students to approach homework problems by searching for similar worked examples and adapting them [15]. There's a trade-off: students may tend to make incorrect generalizations based on syntax but they have also been shown to achieve better long-term retention when studying worked examples [16].

Since learners have a tendency to interpret prototypical examples as isolated instances, instead of representatives of an entire class [17], we elected to create worked examples that would show the procedural solutions to the task while the solver (i) pointed out relevant concepts and connections, (ii) emphasized the decisions made during mathematical modeling, and (iii) reflected on the consequences and inferences that could be drawn from the problem. In essence, the examples modeled the cognition and work of an expert in the field. Pedagogically, we selected problems that exhibited nontrivial entry to the problem, provided an occasion for the professor to externalize his mathematical thinking, and allowed for consideration of the consequences of the solution path or final answer.

To create the narrated worked examples, we used smartpen technology. A smartpen is a ballpoint pen equipped with an internal infrared camera and digital audio recording device. It is used with "digital paper" -- paper covered in a Cartesian array of small dots. The recorded audio is synchornized with flash video of the writing into a "pencast": a sharable flash video. The video shows a light shadow of the writing that is traced in dark green as the video moves forward in time. It is possible to click anywhere on the writing and the audio will begin from that point.

We posted the pencasts on the course website weekly and asked *How do the students respond to the pencasts? How are they using the pencasts?* We collected substantive and technical feedback via surveys on the course website. A total of 107 participants used the pencasts *and* provided feedback (of 251). The results of our mixed-methods inquiry were overwhelming (see [18]): on average, respondents indicated that the pencasts were easy to follow (90%), contained about the right number of steps (79%), were easy to read (90%), and paced about right (86%). For each pencast, we asked the students to self-assess whether they could solve the problem alone, with help, or not at all before watching the pencast and after watching the pencast. The percentage of students who self-assessed as being able to solve the problem after watching each pencast doubled (see Figure 1).

The students most frequently used the pencasts as study aids for summative assessments, which is similar to uses of video solutions reported elsewhere [19]. They reported working on the problem in the pencast or a similar-seeming problem, getting stuck, watching the pencast until they understood, and then continuing to work on the problem. Among students who reported using the pencasts (N = 93), 77.4% used them to study for exams and 64.5% used the to complete homework assignments. They reported using the pencasts in the following ways: to look for key steps to help with other problems (63.4%), to guide them like a solution manual or worked example (52.7%), to coach them through the problem when stuck (36.6%) or as a work-along (33.3%), solely as a substitute for he "back of the book" to check for answers (24.7%), or as a primer or advance organizer for the rest of the problem set (63.4%). These results have implications for the value of the pencasts as instructional aids: lower-level uses of the worked example (e.g., using it as an answer source) are less common than higher-level uses (e.g., to abstract key steps for use in other problems). That is, the students who are watching the pencasts are looking for something *more* than "just an answer," despite the fact that few "answers" were provided in the text.

Responses to our open-ended surveys about how the students liked (or did not like) the pencasts tended to compare them favorably against other worked-example media. For example, in comparing the pencast solutions to static pencil-and-paper solutions, one student commented that it was "easier to see how problems are solved [via pencast] than the book because the explanation isn't in between each of the steps, it's explained through talking." Another student compared an aspect of the pencasts to other solution video media: "you can see everything already there, so you can see where he's going with the solution and not just follow trying to keep up the whole time." The fact that the pencast shows a shadow of the whole solution helps to provide the students foresight in the problem-solving process. As a third example, one student revealed that the pencasts helped to identify similarities among tasks; the pencasts helped me to "realize the places I am getting stuck and then [be] able to solve other problems where I had similar misconceptions."

Over three academic terms, we continue to receive an overwhelmingly positive response from the students in their adoption of and use of this instructional media. We are currently working on linking students' achievement to their use of the pencasts in order to get a sense of how their perceived helpfulness translates to performance goals for the class.

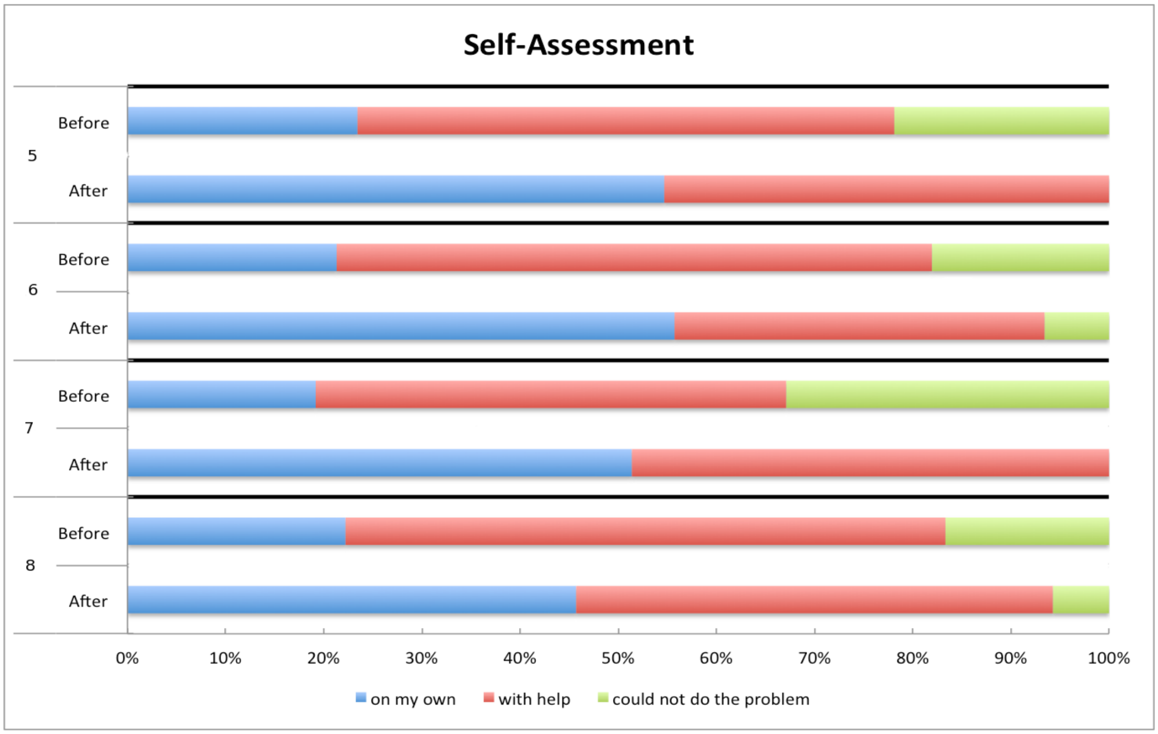

Figure 1: Students' self-assessment of their ability to solve the task independently, before and after watching the pencasts

Anywhere/Anytime Approach

Mathematics classes in the United States typically meet for three to five hours per week for initial instruction and then the students are expected to complete homework assignments consisting of well-chosen problems to work between class meetings. Thus, a great portion of their learning and engagement with the material happens outside the classroom. To supplement book examples, students seek a variety of resources outside their texts and class notes. These typically include Internet resources such as YouTube, Yahoo Answers, or Wolfram Alpha. We hypothesize that the students' search for intellectual support suggests a need for providing quality instructional scaffolding through technology that students may access at their convenience. Some researchers have addressed this out of class need by implementing an inverted classroom model [20]. While we use some resources similar to the inverted classroom model, we have not implemented the full model.

In addition to the pencasts, we began digitizing the classroom through lecture capture. Using SMART sympodia and Notebook software, the course instructor gave interactive lectures and these were captured using Adobe Connect. From the recordings, we were able to create .pdf's of all writing and audio recordings of the classroom interactions (along with a video recording of the SMART sympodium screen capture). Both of these artifacts were posted as resources on the course website for students to access as needed. This practice led to organized, nice-looking lecture presentations that still preserved the option of improvisation (as the use of previously-recorded lectures does not). The tools available with the SMART hardware and software allowed the professor to use simple, but effective, mnemonic and cognitive-load reducing techniques, such as implementing a color-coding system to identify key components to be used in the problem or to highlight different parts of the modeling process. Moreover, the modified slides provided the lecturer with an archived resource of previous lessons (including student questions recorded in the audio stream) that he could revisit and modify in later academic terms. This revisiting allowed for us to better anticipate students' levels of understanding of the content.

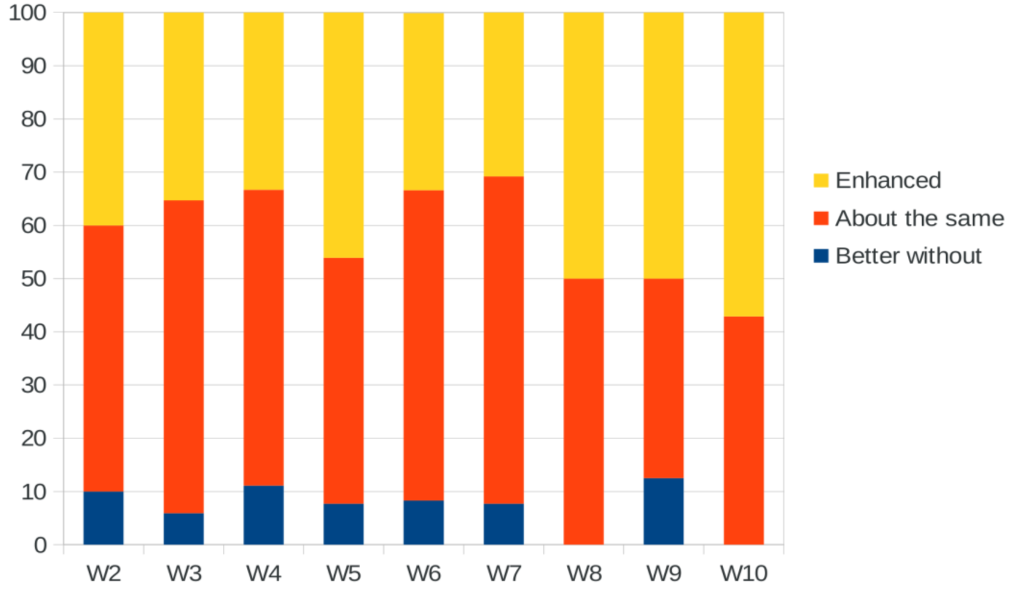

Figure 2. Students’ assessment of if the lecture was enhanced by the use of technology

Another benefit of Adobe Connect and the digitized classroom was the possibility of offering a hybrid course. Adobe Connect, while doing audio and screen capture, is a virtual meeting room which allowed students to connect to and stream the classroom session remotely. This provided students with the option of viewing content synchronously or asynchronously. When we surveyed the students (range: 15 to 31 responses, of maximum 75 enrolled), consistently one-third or more (of the total course enrollment) were watching the videos, suggesting that at least some found them helpful as study aids. There were spikes in video access corresponding with midterms suggesting that the students appreciated having the resource available to them for exam preparation. In addition, the students rated how the examples used during the lesson were executed using technology. There was a mix between responses "enhanced by technology" and "about the same as with no technology," with very few respondents (always fewer than 20%) reporting that the examples were "better without technology" as shown in Figure 2. Students' open-ended responses indicated that they appreciated being able to re-watch lectures while studying, to help with homework, or to fill in gaps in their class notes.

Impact on Mathematics Teaching for Engineering Students

All is not rosy when using technology in the classroom. Certainly, there was a steep learning curve for the professor and his team in becoming accustomed to subtleties in the technology. For example, using the smartpen requires a well-thought out script of the content of the video since it has no editing capabilities aside from splicing and rearranging pages. Similar frustrations are present when using Adobe Connect for lecture capture. Setting up the lecture capture technology each day amounted to a large transactional cost: necessary settings could not be permanently saved to the university computer, three programs needed to be loaded, a microphone needed to be attached and tested, and files from a jump drive needed to be transferred. More importantly, any hiccups in internet connection or glitches in audio equipment caused disruption to the day's lesson or, at times, made the captured video unusable. While Adobe Connect is an excellent (if not always reliable) solution for synchronous meetings, in the future we will move to an offline lecture capture software. We caution that our choice of hardware and software is not optimal and while we were exceptionally pleased with the overall product, we encourage future instructors to seek room- or system-level solutions rather than individual computer-based solutions.

The College of Engineering has begun to strongly encourage all engineering students to take the differential equations course with a modeling approach. Because of our efforts, we now have a dedicated course designation for this differential equations class. We have responded to student feedback and changed the pencasts to better suit their needs. We continue to alter didactic choices during lecture also in response to student feedback on surveys, ensuring that the teacher repeats students' questions and answers (so they are recorded). Students who have taken the class before have returned as tutors for the course. This is an excellent development for two reasons: (i) by virtue of the fact that they want to support the course, it demonstrates that students who have taken the class have found the approach useful in their later engineering studies, and (ii) we can now rely on student-generated explanations of the material, which we hypothesize are closer to how the currently enrolled students think.

In addition, the results of our educational research and the progress of our students are shared, via seminars at the Engineering Education Innovation Center and through informal avenues, with the faculty in the College of Engineering. It is these collaborations among engineering, education, and mathematics disciplines that supported the development of this modeling-based differential equations course.

Recommendations

This project, though small in scale, has been largely successful. We have implemented principles of good teaching practice and incorporate research on students' thinking and learning in the area of differential equations and the engineering disciplines. We have also piloted several technologies and tested their adoptability and efficacy as instructional aids. These activities are possible due to the coordinated efforts of mathematics faculty, engineering faculty, and engineering and mathematics education researchers. Our cooperation was not just about funds, but also about sharing information about our students, about articulating the different disciplines' needs, and about combining results from a variety of discipline-specific research projects.

Multidisciplinary research is not limited to the hard sciences, but is also possible and *indeed necessary* when considering the education of undergraduates. We – mathematics, sciences, engineering, and education – share our students and thus common goals. We must work together in order to do the best for our students and achieve our goals. The take-away message from this report is that these coordinated efforts are possible only when there is communication among departments.

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