# Fluid Mechanics Laboratory Experiment: Calibration of a Semi Circular Weir

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## Abstract

Experiments in open channel flow are important especially to Civil Engineering students. Flow rate is commonly measured in an open channel by using a weir. Weirs that are used in our fluid mechanics lab include a V-notch weir, a suppressed edge weir, and a contracted edge weir. Each of these weirs has been studied extensively, and calibration methods/results have been published in many undergraduate fluid mechanics texts. These weirs are modeled mathematically using an equation relating flow rate to liquid depth over the lowest point of the weir. In an effort to find an additional weir for study, the semi circular weir was proposed. Relating flow rate to liquid depth for this weir involves an infinite series. To make the semi circular weir useful in the lab would require the derivation of a single term, power law equation to relate flow rate to depth. The objective of this study is to derive such an equation, and to present data obtained for purposes of verification. Finding such an equation and avoiding an infinite series permits the use of the semi circular weir, as an additional weir experiment, in the fluid mechanics lab. The results indicate that the semi circular weir is a worthwhile addition to the study of weirs.

## Keywords

Weirs; semi circular weirs; fluid mechanics laboratory; open channel flow

## Introduction

Our fluid mechanics laboratory compliments the lecture course in that the student is able to perform experiments that strengthen lessons learned in the course itself. Experiments include measurements of:

- Viscosity, density, surface tension
- Force exerted on a submerged plane surface
- Flow rate versus pressure loss in various pipeline meters
- Flow rate versus depth for various weirs

Experiments involving weirs are important especially to Civil Engineering students. Weirs that are used include a V-notch weir, a broad crested weir, and a contracted edge weir. Each of these weirs has been studied extensively, and calibration methods/results have been published in many undergraduate fluid mechanics texts. Each of these weirs is modeled mathematically using an equation relating flow rate to liquid depth over the lowest point of the weir. Equations are single term, power law equations[1]. The semi circular weir, however, has received attention, but is not used extensively. The semi circular weir is the subject of this study.

The theoretical equation for flow over a semi circular weir involves a derivation of the relationship between flow rate and depth. Kadlubowski et al.[2] provide equations for what is referred to as overflow devices; these include rectangular, circular, parabolic and triangular shapes. The equation for flow over a semi circular weir is stated, and the solution is expressed in terms of elliptic integrals of the first and second kinds. The efflux of flow over the weir is said to be indeterminate, although a discharge coefficient of 0.59 is stated.

Dodge[3] obtained a theoretical equation for circular sharp crested weirs, but this result is unpublished. The results of Dodge's work have been reported by Stevens[4].

Vatankhan[5] studied circular weirs that would be located in a vertical wall of a channel. The equation given is, again, in terms of elliptic integrals, but can be solved numerically. Vatankhan quotes data obtained by Greve[6], but concludes that there is no simple and accurate theoretical discharge equation in the available technical literature.

Irzooki et al.[7] obtained data for flow over weirs with semicircular openings. Data were obtained for four different radii of openings. These weirs were calibrated against a rectangular sharp crested weir. A dimensional analysis was performed to derive a calibration equation for the semi circular weir. Data from previous studies[4-6] were then quoted and then plotted using the calibration results. Irzooki et al.[7] plots volume flow rate over the weir versus the ratio of depth to cutout diameter. Results for theoretical flow rate include the elliptic integrals mentioned previously.

The objective of this study is to develop an equation for the theoretical discharge of flow over a sharp crested semi circular weir of the same format as that for other weirs. Typically, flow over a weir is described by an equation of the form:

$$Q = C_1 H^n$$

in which $C_1$ and $n$ are constants that are determined by obtaining data on the weir itself,; i.e., calibrating the weir. The format of this equation can be derived by using a binomial series expansion, and avoid evaluating the elliptic integrals obtained in previous studies. This process will allow students to successfully calibrate a semicircular weir in the undergraduate fluid mechanics laboratory.

## Theory

Figure 1 shows a definition sketch of a semi circular weir. The liquid depth above the bottom of the opening is $H$. The radius is $R$, and the $x$-$z$ axes are shown. We define a distance $z$ from the free surface to an element of thickness $dz$. Assuming frictionless flow of an incompressible fluid, the volume flow rate through $dz$ is given by

$$dQ = V\,dA \tag{1}$$

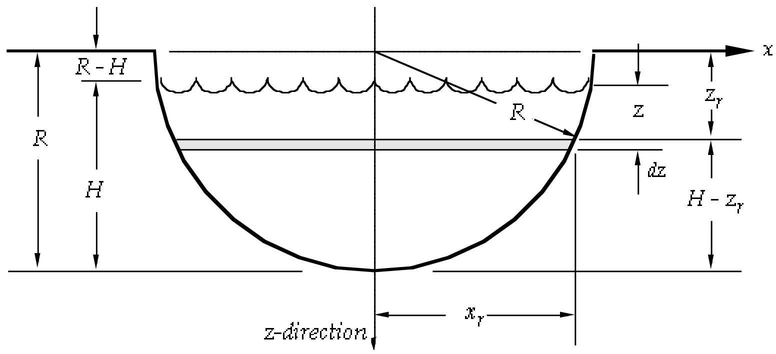


**Figure 1.** *Cross section of a semi circular weir.*

Applying Bernoulli's equation to the cross section gives for the velocity

$$V = (2gz)^{1/2} \tag{2}$$

The area of the element is

$$dA = 2x_r\, dz \tag{3}$$

The distance $x_r$ may be obtained from:

$$x_r^2 + z_r^2 = R^2$$

or

$$x_r^2 = R^2 - z_r^2 \tag{4}$$

and $z_r$ is found with

$$z_r = z - R - H \tag{5}$$

Substituting Equations 2–5 into 1 gives

$$dQ = V\, dA = (2gz)^{1/2}\, (2x_r\, dz)$$

or

$$dQ = 2(R^2 - (z + R - H)^2)^{1/2}\, (2gz)^{1/2}\, dz$$

Integrating this equation from 0 to $Q$, and correspondingly from 0 to $H$, we write

$$Q = \int_0^H 2(R^2 - (z + R - H)^2)^{1/2}\, (2gz)^{1/2}\, dz$$

Factoring the constants,

$$Q = 2(2g)^{1/2} \int_0^H z^{1/2} [R^2 - (z + R - H)^2]^{1/2} )dz$$

Factoring radius $R$ from the integrand, we obtain

$$Q = 2R(2g)^{1/2} \int_0^H z^{1/2} \left[1 - \frac{(z + R - H)^2}{R^2}\right]^{1/2} dz \tag{6}$$

Continuing further requires the integrand to be changed into another form. Typically, we would encounter elliptic integrals. However, the expression in square brackets may be rewritten using a binomial series expansion (Newton's binomial theorem):

$$(1 - x^2)^{1/2} = 1 - \frac{x^2}{2} - \frac{x^4}{8} - \frac{x^6}{16} \cdots$$

The term in square brackets then is written as

$$\left[1 - \frac{(z + R - H)^2}{R^2}\right]^{1/2} = 1 - \frac{(z + R - H)^2}{2R^2} - \frac{(z + R - H)^4}{8R^4} - \frac{(z + R - H)^6}{16R^6} \cdots \tag{7}$$

The second term in the series is expanded with minor effort:

$$\frac{(z + R - H)^2}{2R^2} = \frac{z^2 + 2Rz - 2Hz + R^2 - 2HR + H^2}{2R^2}$$

Then,

$$1 - \frac{(z + R - H)^2}{2R^2} = 1 - \frac{z^2}{R^2} - \frac{z}{R} + \frac{Hz}{R^2} - \frac{1}{2} + \frac{H}{R} - \frac{H^2}{2R^2}$$

or

$$1 - \frac{(z + R - H)^2}{2R^2} = \frac{1}{2} - \frac{z^2}{R^2} - \frac{z}{R} + \frac{Hz}{R^2} + \frac{H}{R} - \frac{H^2}{2R^2}$$

Substituting this result into Equation 7,

$$\left[1 - \frac{(z + R - H)^2}{R^2}\right]^{1/2} = \frac{1}{2} - \frac{z^2}{R^2} - \frac{z}{R} + \frac{Hz}{R^2} + \frac{H}{R} - \frac{H^2}{2R^2}$$

We recall that this result includes only the first two terms of the series in Equation 7. We continue by substituting into Equation 6, which can now be integrated term by term:

$$Q = 2R(2g)^{1/2} \int_0^H z^{1/2} \left(\frac{1}{2} - \frac{z^2}{R^2} - \frac{z}{R} + \frac{Hz}{R^2} + \frac{H}{R} - \frac{H^2}{2R^2}\right) dz$$

$$Q = 2R(2g)^{1/2} \int_0^H \left( z^{1/2}\frac{1}{2} - \frac{z^{5/2}}{R^2} - \frac{z^{3/2}}{R} + \frac{Hz^{3/2}}{R^2} + z^{1/2}\frac{H}{R} - z^{1/2}\frac{H^2}{2R^2} \right) dz$$

$$Q = 2R(2g)^{1/2} \left( z^{3/2}\frac{1}{3} - \frac{2z^{7/2}}{7R^2} - \frac{2z^{5/2}}{5R} + \frac{2Hz^{5/2}}{5R^2} + z^{3/2}\frac{2H}{3R} - z^{3/2}\frac{H^2}{3R^2} \right)_0^H$$

Substituting the upper limit,

$$Q = 2R(2g)^{1/2} \left( H^{3/2}\frac{1}{3} - \frac{2H^{7/2}}{7R^2} - \frac{2H^{5/2}}{5R} + \frac{2H^{7/2}}{5R^2} + \frac{2H^{5/2}}{3R} - \frac{H^{7/2}}{3R^2} \right)$$

Combining terms with like exponents,

$$Q = 2R(2g)^{1/2} [H^{3/2} (1/3) + H^{5/2} (2/3 - 2/5) + H^{7/2} (2/5 - 2/7 - 1/3)]$$

which becomes

$$Q = 2R(2g)^{1/2} \left( \frac{H^{3/2}}{3} + \frac{4H^{5/2}}{15} - \frac{23H^{7/2}}{105} \right)$$

Using only the first two terms in the binomial series, this equation has the following form:

$$Q = C_1H^{3/2} + C_2H^{5/2} + C_3H^{7/2} \tag{8}$$

where the $C_i$ values are constants. Higher order powers of $H$ will surface if more terms are used. The objective now becomes finding values for the constants, and this will depend on data obtained in the laboratory. In what follows, results of an experiment will be presented to determine the calibration constants for those that appear in Equation 8.

## Experimental Procedure

Figure 2 is a sketch of the apparatus used in obtaining data. It consists of a tank of water flanked on both sides with centrifugal pumps. Each pump discharges into a pipe containing a turbine meter and a flow control valve. Water is pumped into a head tank and flows underneath the head gate into the flow channel. The channel width is 12 in and it is 15 ft long. Water travels over the weir and makes its way back to the sump tank. The apparatus was manufactured by Engineering Lab Design (eldinc.com). The flow meters were manufactured by Onicon (onicon.com) and have been calibrated. The weir has been manufactured in-house, and has a radius of 9.125 in. Depth measurements far upstream of the weir were made with a point gage supplied with the open channel device.

## Results

Sixty-nine data points of flow rate versus depth were obtained, and are graphed in Figure 3. The horizontal axis is the depth in ft, and the vertical axis is water flow rate in gallons per minute.

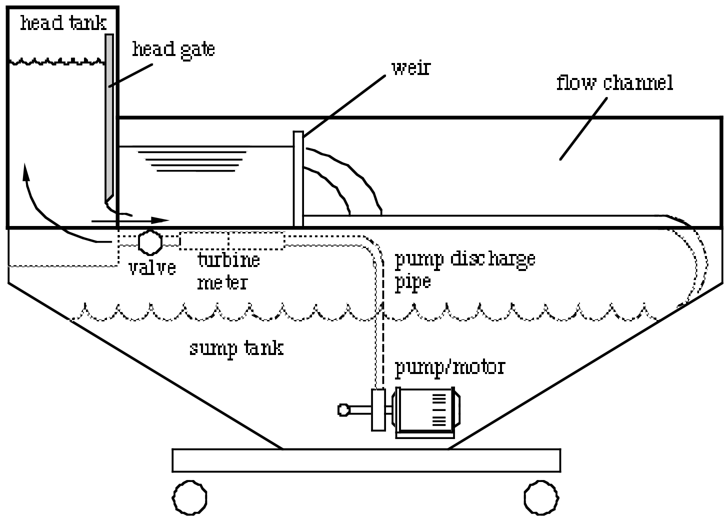


**Figure 2.** *Schematic of the apparatus used in this study.*

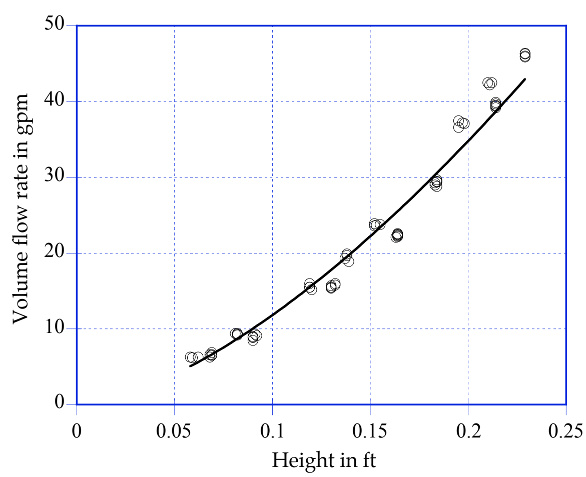


**Figure 3.** *Data points; flow rate over a semi circular weir as a function of upstream water height.*

The best fit curve of these data are:

$$Q = 426.8\, H^{1.55} \qquad R^2 = 0.9804 \tag{9}$$

## Educational Value

The open channel flow device in the fluid mechanics laboratory was supplied by the manufacturer with several different weirs: V-notch, contracted edge, and suppressed edge. The calibration equations for these weirs have the same form, namely a constant x height to a power:

V-notch $\qquad Q_{ac} = C_{vn} C_1 H^{5/2} = C H^{5/2}$

Contracted edge $Q_{ac} = C_c C_1 H^{3/2} = CH^{3/2}$

Suppressed edge $Q_{ac} = C_s C_1 H^{3/2} = CH^{3/2}$

The constants $C_{vn}$, $C_c$ and $C_s$ are discharge coefficients that take into account frictional effects not included in the theoretical equations. The product of the discharge coefficient and the constant $C_1$ (= $C$) for each weir are to be determined by experiment. Thus, student groups would obtain many data points on actual flow rate ($Q_{ac}$) and height ($H$), and then determine the value of the constant $C$ in the preceding equations.

The addition of another weir to this ensemble is thought to be most beneficial to those students taking fluid mechanics and those taking open channel hydraulics. So this study was undertaken in order to determine whether flow over a semi circular weir could be added to the lab. What precluded its addition heretofore was the theoretical equation for this weir, which involved an infinite series. Using merely two terms of this series yielded results involving the height raised to the 3/2, 5/2 and 7/2 power (Equation 8):

$$Q_{ac} = C_{sc}(C_1 H^{3/2} + C_2 H^{5/2} + C_3 H^{7/2}) \qquad (8)$$

If data on the semi circular weir would indicate that any of these terms fit the results appropriately, then this weir could be added without difficulty. The results indicate that the actual flow rate equation is:

$$Q_{ac} = CH^{3/2}$$

Although not demonstrated here, further manipulation of the data shows that the coefficients of $H^{5/2}$ and $H^{7/2}$ reduce to zero very quickly.

## Uncertainty Analysis

A detailed uncertainty analysis was not performed for this study, although the data obtained can be used to perform one. This is left to the students who are taking the lab. However, depth of liquid was measured with a depth gage, whose measurements are to the nearest 16th of an inch. The flow rate was determined with a turbine-type meter installed in the flow line, which was calibrated by the manufacturer to within 5% of the mean value.

## Conclusions

The objective of this study was to obtain a relationship for flow rate over the semi circular weir in terms of the upstream height. It was desired to determine a single term equation such as Equation 9. The mathematical model for flow over the weir yielded several terms, all expressed as the product of a constant and the liquid height upstream of the weir. The method presented indicates that the exponent of the result is an integral multiple of 3/2 (approximately). The resultant equation allows the student to obtain data and derive an equation without resorting to elliptic integrals or an infinite series. This approach is ideal for the undergraduate fluid mechanics lab.

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## William S. Janna

William S. Janna joined the faculty of The University of Memphis in 1987 as Chair of the Department of Mechanical Engineering. He served as Associate Dean for Graduate Studies and Research in the Herff College of Engineering. His research interests include boundary layer methods of solution for various engineering problems, and modeling the sublimation of ice objects of various shapes. He is the author of three textbooks, and teaches continuing education courses in the area of piping systems and in heat exchanger design and selection, for ASME. Dr. Janna received a B.S. degree, an M.S.M.E. and a Ph.D. from the University of Toledo.

## Paul Palazolo

Paul Palazolo joined the faculty of The University of Memphis in 1994 as the Assistant Dean for Recruiting, and as a faculty member in the Department of Civil Engineering. He also served as the Associate Dean for Outreach, Recruiting, and Retention until January 2014. He has served as an officer in various positions in the Southeast Section of ASEE, and as a local and national officer of ASEE. His interests are first year experiences in engineering, and education in engineering fundamentals. He was a registered professional engineer until 2014 when he retired his license. Dr. Palazolo received B.S. and M. S. degrees from Memphis State University, and a Ph.D. from Georgia Tech.

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