Uncertainty Analysis of Thermal Conductivity Measurement by a Homemade Apparatus

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Abstract

In 2008 a homemade apparatus was designed and built to determine the conductivity of a variety of metal test samples. It has been in use every semester since then in the Heat Transfer Lab for Mechanical Engineering Technology (MET) students at Penn State Erie. The device is specifically designed and built for this purpose, and it provides repeatable data when used for its intended function. The design of the device is similar to a device that is used for the measurement of conductivity of thermal interface materials in industry. It is now being used as a research tool to try to measure the conductivity of plastic and composite materials.

Since this device is being used for an unintended purpose it was decided that we should take a closer look at possible sources of error and improvements that might be made to mitigate some of those errors. This paper presents a brief overview of the basic function of the device, including a qualitative discussion of some of the likely sources of error. Then, it includes a discussion of the procedure for measuring the conductivity of plastic and composite materials, also with a qualitative discussion of likely sources of error. The main purpose of the paper is to present a quantitative uncertainty analysis of the thermal conductivity measurement for both the metal samples and for the plastic and composite materials. The paper concludes with thoughts on possible changes that could be made to improve the accuracy of the existing device.

Introduction

In 2008 a homemade apparatus [1] was designed and built to determine the conductivity of a variety of metal test samples. It has been in use every semester since then in the Heat Transfer Lab for Mechanical Engineering Technology (MET) students at Penn State Erie. The device is specifically designed and built for this purpose, and it provides repeatable data when used for its intended function. Even though the results are repeatable, they are not particularly accurate. However, the conductivities found for the metals are close enough for the students in an undergraduate MET course in heat transfer to be able to learn from the exercise. Possible sources of error can be discussed in class, which helps to enhance the students understanding of the overall exercise.

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Since this device is being used for an unintended purpose it was decided that we should take a closer look at possible sources of error and improvements that might be made to mitigate some of those errors. This paper presents a brief overview of the basic function of the device, including a qualitative discussion of some of the likely sources of error. Then, it includes a discussion of the procedure for measuring the conductivity of plastic and composite materials, also with a qualitative discussion of likely sources of error. The main purpose of the paper is to present a quantitative uncertainty analysis of the thermal conductivity measurement for both the metal samples and for the plastic and composite materials. The paper concludes with thoughts on possible changes that could be made to improve the accuracy of the existing device.

Experimental Setup:

Measurement of the thermal conductivity of cylinder shaped specimen

The apparatus for the measurement of thermal conductivity is shown in Fig. 1. Two cylinders are pressed together by a spring. A heater is embedded in the top cylinder, which is 6061 aluminum. The cooling block is attached to the bottom cylinder, which is the test specimen. There are five specimens currently available, which are aluminum, brass, bronze, cast iron, and stainless steel. The cooling block has a channel for city water to pass through and cool down the block. The cylinder diameter is 1.25 inches. The power of the heater is controlled by a variable autotransformer and measured by a wattmeter. There are four T-type thermocouples embedded along the axis of two cylinders. The distance between thermocouples T_1 and T_2 is 3 inches. The distance between T_2 and the interface surface is 0.38 inches. The distance between T_3 and the interface surface is 0.50 inches. The distance between thermocouples T_3 and T_4 is 3 inches. A National Instruments DAQ unit is connected to the thermocouples and a LabVIEW program is used to collect the readings. The top and bottom cylinders are covered by foam insulation. The thermal conductivity of the test specimen is determined from the measured data.

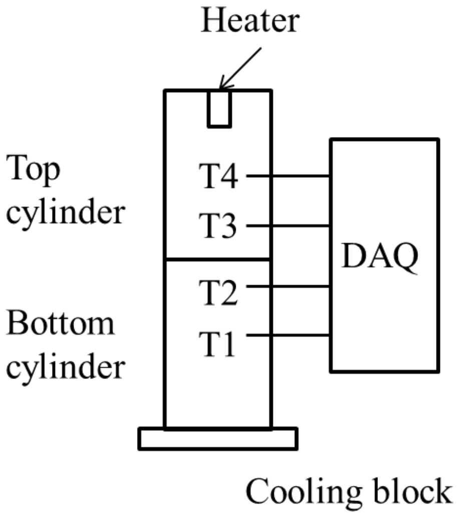

Figure 1. Apparatus for the measurement of thermal conductivity.

The heat flux along the axis of the cylinder can be calculated by

$$q'' = -k\frac{dT}{dx} \qquad (1)$$

where k is the thermal conductivity of the cylinder. The heat flux along the axis of the top cylinder can be calculated by

$$q'' = k_t\frac{T_4 - T_3}{L_t} \qquad (2)$$

where k_t is the thermal conductivity of the top cylinder, which is 180 W/mk and L_t is the distance between thermocouples T_3 and T_4. Neglect the heat loss from the cylinders to the surrounding air, the heat flux along the axis of the top cylinder equals to the one through the bottom cylinder. So the thermal conductivity of the bottom cylinder can be calculated by

$$k_b = \frac{q'' L_b}{T_2 - T_1} \qquad (3)$$

where L_b is the distance between thermocouples T_1 and T_2.

Uncertainty analysis of the thermal conductivity of cylinder shaped specimen

Based on an uncertainty analysis method described by Kline and McClintock [2], the uncertainty of q'' can be calculated by

$$u_{q''} = \sqrt{(\frac{\partial q''}{\partial T_4} u_{T_4})^2 + (\frac{\partial q''}{\partial T_3} u_{T_3})^2 + (\frac{\partial q''}{\partial T_t} u_{L_t})^2} \qquad (4)$$

where u_{T_3}, $u_{T_4}$and u_{L_t} are the uncertainty of T_3 , T_4 and L_t, respectively. The uncertainty of k_b can be calculated by

$$u_{k_b} = \sqrt{(\frac{\partial k_b}{\partial T_1} u_{T_1})^2 + (\frac{\partial k_b}{\partial T_2} u_{T_2})^2 + (\frac{\partial k_b}{\partial q''} u_{q''})^2 + (\frac{\partial k_b}{\partial L_b} u_{L_b})^2} \qquad (5)$$

where u_{T_1}, u_{T_2}, $u_{q''}$ and u_{L_b} are the uncertainty of T_1 , T_2 , q'' and L_b, respectively.

Measurement of thermal conductivity of plastic and composite materials

The apparatus for the measurement of thermal conductivity of plastic and composite materials is the same as the one for the measurement of thermal conductivity, which is shown in Fig. 1. Both the top and bottom cylinder are 6061 aluminum. For this test, the specimen can’t be very thick due to the limitation of the apparatus. The graphite specimen is chosen to represent the plastic and composite materials. The graphite specimen is sandwiched between the top and bottom cylinder, which is shown in Fig 2. The top interface surface temperature is T_a and the bottom interface surface temperature is T_b. The graphite specimen has a thickness of 5mm and a diameter of 1.13 inches. Fig.3 shows the 5mm graphite specimen sandwiched between the top and bottom cylinder. During experimentation, the entire system is monitored at a specified condition to ensure a true steady state. When the temperature fluctuation in each measured point in the system is less than 0.5 ^{o}C in 30 min, steady state is assumed to be reached.

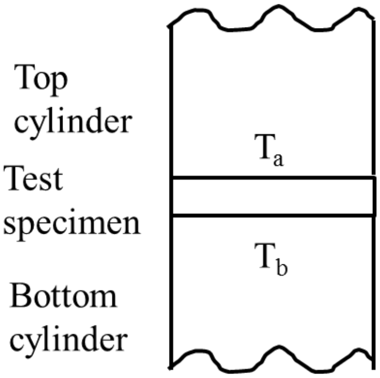

Figure 2. Diagram of the graphite specimen sandwiched between the top and bottom cylinder.

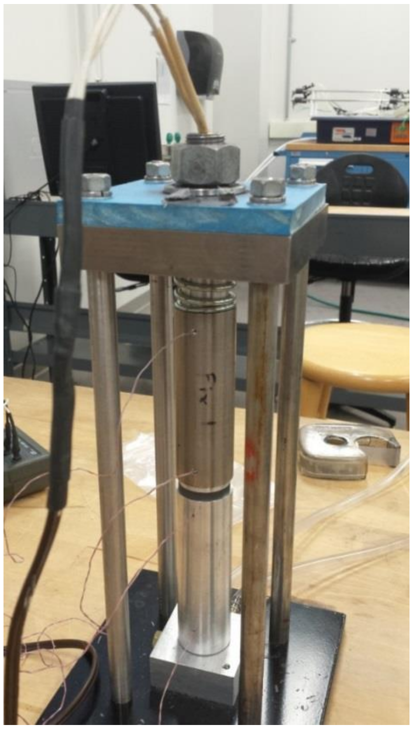

Figure 3. 5mm graphite specimen is sandwiched between the top and bottom cylinder.

The heat flux along the axis of the top cylinder can be calculated by

$$q''_t = k_t \frac{T_4 - T_3}{L_t} \qquad (6)$$

The heat flux along the axis of the bottom cylinder can be calculated by

$$q''_b = k_b \frac{T_2 - T_1}{L_b} \qquad (7)$$

The heat flux along the axis of the graphite specimen can be calculated by

$$q''_m = \frac{q''_t + q''_b}{2} \qquad (8)$$

The top interface surface temperature can be calculated by linear extrapolation

$$T_a = T_3 - L_a \frac{T_4 - T_3}{L_t} \qquad (9)$$

where L_a is the distance between T_3 and the interface surface, which is 0.50 inches.

The bottom interface surface temperature can be calculated by linear extrapolation

$$T_b = T_2 + L_c \frac{T_2 - T_1}{L_b} \qquad (10)$$

where L_c is the distance between T_2 and the interface surface, which is 0.38 inches.

Neglecting the thermal contact resistances between the cylinders and the graphite specimen, the thermal conductivity of the graphite specimen can be calculated by

$$k_m = \frac{q''_m L_m}{T_a - T_b} \qquad (11)$$

where L_m is thickness of the graphite specimen.

The uncertainty analysis of the thermal conductivity of the graphite specimen is conducted using the method described by Kline and McClintock [2].

Result and Discussion

Edwards's experiment [1] of aluminum cylinder at the bottom showed that T_1, T_2,T_3 and T_4 were 38.1 °C, 44.5 °C , 58.9 °C and 65.5 °C, respectively. Using Eqn. 2 and 3, the measured thermal conductivity of the aluminum was 185.6 W/mk, which is very close to the theoretical value of 180 W/mk. The percentage of error is only 3.1%. The uncertainty of temperature measurement from the thermocouples is 0.5 °C, which is the half value of the thermocouple tolerance value of 1°C. The uncertainty of distance measurement from the caliper is 0.0005 inch, which is the half value of the caliper resolution of 0.001 inch. Using Eqn. 4 and 5, the uncertainty of the measured thermal conductivity of the aluminum was 28.6 W/mk. The major part of this uncertainty value comes from the uncertainty of the temperature measurement. So using high precision thermocouples, which reduces the uncertainty of the temperature measurement, will significantly reduce the uncertainty of the measured thermal conductivity of the aluminum.

Experiment of graphite specimen showed that T_1, T_2, T_3 and T_4 were 49.0 ^{o}C, 57.1 ^{o}C , 73.7 ^{o}C and 81.6 ^{o}C, respectively. Using Eqn. 6 to 11, the measured thermal conductivity of the graphite was 6.7 W/mk, which is very different to the theoretical value of 100 W/mk. The main reason is that the thermal contact resistances between the cylinders and the graphite specimen are neglected. Thermal grease should be applied between the graphite specimen and the cylinders to reduce the thermal contact resistance. The uncertainty of the measured thermal conductivity of the graphite was 0.57 W/mk.

Conclusions

This paper presented the uncertainty analysis for the thermal conductivity of a homemade apparatus. This apparatus can measure the thermal conductivity of cylinder shaped material very accurately. The uncertainty of the measurement can be improved by using high precision thermocouples. The preliminary test of a graphite specimen is not very successful because of the neglect of thermal contact resistance. The uncertainty analysis of the graphite specimen is also conducted.

References

[1] R. Edwards, "A Homemade 2 Dimensional Thermal Conduction Apparatus Designed As A Student Project", *2008 Annual Conference & Exposition, Pittsburgh, Pennsylvania, 2008, June*. ASEE Conferences, 2008. https://peer.asee.org/3222 Internet. 11 Jan, 2018

[2] S. J. Kline, F.A. McClintock, "Describing Uncertainties in Single-sample Experiments", Journal of Mechanical Engineering, 75, 3-8, 1953