Introduction to Quantum Control with a Focus on Measurement Feedback Quantum Control

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Abstract—Thermostats, washing machines, robotic systems, and many other consumer and industrial products are regulated with control theory to automate their tasks. These devices can follow prescribed routines as well as compensate for environmental fluctuations through sensing of variables of the system. However, traditional control theory, which often involves taking measurements of relevant quantities in order to correct deviations from the desired behavior, does not transfer directly to quantum systems. Quantum systems may be as small as a single hydrogen atom and are comprised of subatomic particles subject to non-negligible quantum effects due to their small size. These systems are inherently affected by the action of taking a measurement through Heisenberg's Uncertainty Principle. Noise is introduced into the system as the increase in certainty of one variable caused by a measurement necessarily decreases the certainty of a complementary variable's state. While open-loop quantum control, which avoids taking measurements, is similar to traditional open-loop control, closed-loop quantum control is very different than classical closed-loop control because of the noise created by taking measurements. Approaches that are currently being refined for closed-loop quantum control include closed-loop learning control, direct feedback control, indirect feedback control, quantum filtering, and coherent quantum feedback. These methods provide ways to control some quantum systems while more methods and alterations are being created to work on other types of quantum systems. Many of these quantum control techniques are inspired by similar approaches in traditional control but are altered to apply to quantum systems. Quantum control aims to provide ways of controlling all types of quantum systems through certain combinations of new and old control strategies. This paper provides an introductory survey of the current state of quantum control and the progress made to this point.

I. Introduction

The early 1900s brought many changes with the derivation of quantum mechanics and a new interpretation of particles at the microscopic level. Some systems are called quantum systems as the constituents of these systems, called particles, undergo nonnegligible quantum effects, due to their small size, including but not limited to entanglement, tunneling, and wave-particle duality[1]. It is important to realize the difference between open and closed quantum systems. Open quantum systems are open to the environment surrounding them and thus to the noise introduced by that environment. Closed quantum systems are said to be unaffected by their surroundings. The desire to control these quantum systems has led to the field of quantum control. Quantum control can be helpful for many applications. While the current applications are mostly focused on improving scientific research, future research may yield applications that affect the general public.

Quantum control has many areas in which it has yet to be fully developed. These areas must be developed similarly to proven traditional control areas by combining control theory with the mathematics of quantum mechanics. This is a nontrivial process that has taken years and scores of scientists to develop the currently functioning methods of quantum control. In addition to manipulating the two fields of traditional control theory and quantum mechanics into a unified theory, Heisenberg's Uncertainty Principle implies that noise is injected into the system for closed-loop quantum control. This principle says that taking a measurement provides certainty of one variable which must decrease the certainty of a complementary variable. So, closed-loop quantum control, which employs the usage of measurements, is restricted in the knowledge of two related variables. This extra noise in the equations makes the closed-loop quantum control derivations more complex.

A similar survey paper by Dong and Petersen overviews quantum control and its purposes[1]. The article first examines the bilinear model which has three equations derived from the Schrödinger time-dependent equation. These equations model an evolving system and can lead to control of the system as proven in "molecular systems in physical chemistry and spin systems in [nuclear magnetic resonance]" [1]. Furthermore, it explains that Markovian master equations can be used to model the evolution of an open quantum system when there is a "short environmental correlation time . . . and memory effects may be neglected"[1]. However, stochastic master equations are used for quantum systems that must be continuously measured to create feedback for the controller. In addition, linear quantum stochastic differential equations can be used to describe some systems, such as linear quantum optics, but must have constraints to ensure they are physically realizable. These equations are derived to model quantum systems and their evolution over time to create control methods.

Mabuchi and Khaneja wrote an article evaluating the progress in applications of quantum control discussed during a Principles and Applications of Control in Quantum Systems (PRACQSYS) workshop[2]. In particular, control theory is being used to improve magnetic resonance, control over atomic physics, computer miniaturization, and other fields.

Mabuchi and Khaneja describe how modern nuclear magnetic resonance (NMR) orients the magnetic moments of nuclei using a large magnet[2]. When the magnet is switched off, the nuclei are solely responsible for the magnetic field. This field is examined by analyzing a nearby coil's frequency using a Fourier transform to yield its peak frequency, the Larmor frequency. These frequencies, established for many nuclei, allow for the structure of proteins to be determined. Multidimensional NMR can make a spectrum of Larmor frequencies associated to coupled spin pairs. Control theory is now being used to create "time-optimal pulse sequences that induce a certain evolution of coupled spins or reach a target state with minimum relaxation losses"[2].

The paper also describes how atomic physics owes new developments to control theory's application to trapped ions and single-atom cavity electrodynamics[2]. Trapped ions now can be shown to represent an elementary quantum model coupled with harmonic oscillators. They have had their quantum states successfully manipulated and provide sophisticated open-loop control but are difficult to work with for real-time monitoring. They are being used to measure frequencies and process quantum information. Single-atom cavity quantum electrodynamics can also be controlled but allow for continuous measurements to create real-time feedback.

The PRACQSYS workshop also included discussion on the utilization of electrons' spin for carrying information rather than their charge[2]. This development could increase computer speeds and decrease their size. However, a major challenge in using electron spin for carrying information is quickly changing an electron's quantum state without disturbing neighboring electrons. Also, interaction of a quantum system with its environment leads to relaxation and loss of signal. In the process of optimizing these relaxed quantum states, constraint bilinear control problems are now being investigated[2]. Another discovery has found that quantum filtering equations have now created real-time feedback control on "single-atom cavity [quantum electrodynamics] and hyperfine spin dynamics"[2]. These equations also show promise for controlling other quantum systems. Future research holds implications for controlling a variety of systems in a wide variety of applications.

The rest of the paper is organized as follows: Section II describes the history and categorization of quantum control. Section III explains the models currently guiding measurement feedback quantum control. Section IV analyzes some of the results recently found in measurement feedback quantum control. Section V concludes the paper.

II. Background

Quantum control is a product of both quantum mechanics and control theory. Classical control began centuries ago and can be seen in devices as early as the centrifugal governor in James Watt's 1788 steam engine. Quantum mechanics has been a revolutionary evolution of ideas generated in the early 20th century. These ideas have successfully matched experiments and have been applied to various subjects. The two fields merged into quantum control starting around 1950. Quantum control theory is still an active area of research. The best strategies to account for quantum effects is an open problem.

Quantum control, like classical control, is split into open-loop control and closed-loop control. Each of these categories employs different mathematical techniques and equations. Open-loop control uses a controller without feedback measurements to augment the quantum system. While open-loop quantum control has had success, it has a limited range of applications. Open-loop strategies include Lyapunov-based control, variable control, and incoherent control[1]. Closed-loop control involves taking a measurement of the system. In quantum control, this introduces some noise into the system and has made the progress slower in this field. Strategies include closed-loop learning control, direct feedback control, indirect feedback control, and quantum filtering[1,3].

A. Open-loop quantum control

The principles of open-loop control are very similar for classical and quantum systems[3]. The similarities are due to the fact that there are no measurements taken of the plant, see Figure 1.

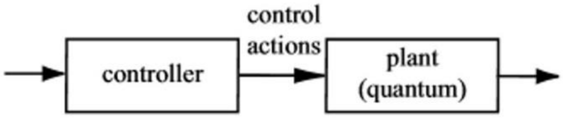

Figure 1: Open-loop quantum control[1]

Derivations of quantum open-loop control start with the time-evolution operator of the quantum system: the Schrödinger equation for the wave function, the Liouville equation for the density matrix, or other equations that model the dynamic evolution of the system[3]. From these equations, theory is developed very similarly to classical control derivations so that the desired target value can be achieved. Open-loop strategies, including coherent and incoherent control approaches, optimal control techniques, and Lyapunov methods, have had great success but have limited applications[1].

In addition, open-loop processes have also been able to implement some closed-loop strategies. For example, Lyapunov-based control methods are powerful tools for feedback controller design in classical control theory[1]. Because measurement of quantum systems, as is performed in closed-loop control, tends to destroy the measured state, feedback control design is first completed by a computer simulation. Then, this simulation yields a control sequence which can then be applied to the actual plant in an open loop. This strategy is a way to use a closed-loop strategy through an open-loop architecture and is very effective for some difficult quantum systems.

B. Closed-loop quantum control

Closed-loop control has several different classical approaches that are now being translated to quantum control. These approaches include closed-loop learning control and quantum feedback control[1]. Closed-loop learning control observes a sample and attempts to control it. Then, a learning algorithm derives a better approach based on this attempt, and the improved control method is then implemented on a newly prepared sample. After a few iterations, this type of quantum system can be controlled with increased efficiency.

Quantum feedback control is similar to normal feedback control as it has been proven to better regulate quantum systems than through open-loop control. It also deals with the decoherence that arises in quantum systems that are not completely isolated. Quantum feedback control has two types: coherent quantum feedback control and measurement feedback quantum control. Coherent feedback control uses quantum information from the quantum plant in order to affect a quantum controller which then regulates the quantum plant, as illustrated in Figure 2.

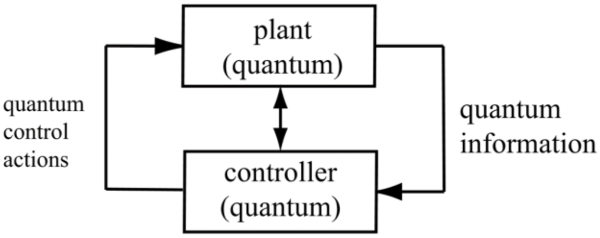

Figure 2: Coherent feedback control[1]

Measurement feedback quantum control uses a measurement as classical information from the quantum plant to regulate a classical controller to then regulate the quantum plant; see Figure 3.

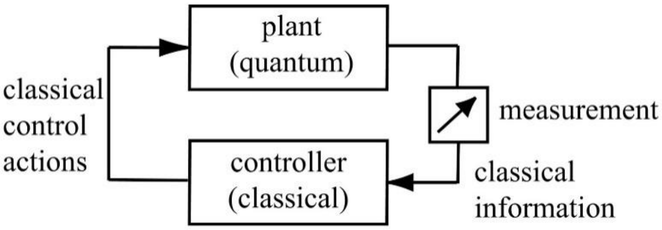

Figure 3: Measurement feedback quantum control[1]

Quantum feedback control typically consists of either projective or continuous, weak measurement[1]. Quantum system identification or state estimation problems are a prerequisite for quantum feedback control as the knowledge of a system's model and its states are needed. For a few systems, quantum nondemolition measurements can be made which do not affect the observable state by not changing the system. Also, continuous measuring of a changing variable is achieved through quantum filtering theory, which reduces the noise caused by the measurements. Feedback can then be implemented by a Hamiltonian feedback controller which affects the system's evolution. Feedback Hamiltonians have been successfully demonstrated with Markovian and Bayesian quantum feedback on a variety of systems. Quantum feedback control has improved control of "squeezed states, quantum entanglement, and quantum state reduction in many areas such as quantum optics, superconducting quantum systems, Bose-Einstein condensate and nanomechanical systems"[1].

Experiment has shown that closed-loop quantum control can be more successful than open-loop quantum control. One experiment focuses on controlling the joint spin-state of two cesium atoms in an optical cavity[4]. Their states differ by their differing cavity transmission levels. The cavity transmission is measured by a single photon counter, which is connected to a digital signal processor (DSP) controlling the intensities of both a repumping laser and a depumping laser in real time; see Figure 4.

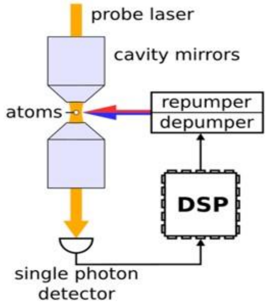

Figure 4: Experimental Setup[4]

This experiment is an example of measurement feedback quantum control as the single photon detector takes measurements of the quantum system of the two cesium atoms in an optical cavity and sends this as classical information into the digital signal processor, which then regulates the quantum plant by means of a repumper and a depumper[4]. This closed-loop process was found to control the system with 84% accuracy. In comparison, an open loop utilizing a weak, continuous repumping laser was only able to achieve 33% accuracy. Also, an open loop using an optimal repumping rate was only able to achieve 37% accuracy.

III. Foundational Models of Measurement Feedback Quantum Control

Theoretical research in measurement feedback quantum control is mostly focused around two-level atoms. A two-level atom is an atom with two possible states for the electron: ground state or excited state. These two-level atoms are used for modeling the efficacy of different strategies because they are the simplest nonlinear quantum systems.

One article examines different attempts to control a “continuously monitored open quantum system”[5]. To incorporate quantum feedback in an open quantum system, “information lost from the system into that environment [has to be engineered] to affect the system again”[5]. This measurement feedback quantum control loop has “a quantum system [consisting of a two-level atom], a classical detector (which turns quantum information into classical information), and a classical actuator (which uses the classical information to affect the quantum system)”[5].

Two major strategies in measurement feedback quantum control are Bayesian feedback and Markovian feedback[5]. Bayesian feedback provides feedback based on its estimate of the system’s current state. This estimate is constructed from the knowledge of several previous states of the system. Bayesian feedback allows the state estimate ρ_I to be expressed in a stochastic master equation and is shown as (1).

$$d\rho_I = dt[\mathcal{L} + \mathcal{U}]\rho_I - i[F(t, \breve{\rho}_I), \rho_I] \tag{1}$$

Note that (1) requires an estimate of the system, $\breve{\rho}_I$.

Markovian feedback was first introduced in quantum optics by Wiseman and Milburn in 1993[5]. It uses a measurement to immediately alter the system’s state without filtering the signal.

Markovian feedback is guided by use of the Wiseman-Milburn feedback master equation and is shown as (2).

$$\dot{\rho} = \mathcal{L}_0\rho + \mathcal{D}[c]\rho - i\sqrt{\eta}[F, cp + pc^\dagger] + \mathcal{D}[F]\rho \qquad (2)$$

Thus, both Bayesian feedback and Markovian feedback have equations to sufficiently describe and control the system.

IV. Analysis of Measurement Feedback Quantum Control

Bayesian feedback has been shown to be superior to Markovian feedback in theory. In fact, Wiseman et al. claim that “Bayesian feedback never performs worse than Markovian feedback”[5]. This is the same as the classical result and stems from the same explanation: Bayesian feedback utilizes more information of the system. However, it will be difficult to determine which method is more practical because of the difficulty associated with implementing Bayesian feedback. There is also the question of the negative impact due to imperfections of the loops. These imperfections have been shown to typically have small repercussions for Markovian feedback but have not been studied for Bayesian feedback. The imperfections have the opportunity to discredit the statement that “Bayesian feedback never performs worse than Markovian feedback” through future research[5].

Doherty and Jacobs present an article that shows how to use feedback in quantum systems with continuous measurements[6]. They summarized their results by saying “that when using feedback by estimation, and when we average over the conditional evolution, there is an additional uncertainty . . . due to measurement inefficiency, and that this excess uncertainty decreases with the magnitude of the feedback”[6]. They also explain that the uncertainty is from “the noise which is continually fed into the system as the result of the measurement” to explain the correlation between the uncertainty and the feedback magnitude[6]. However, Doherty and Jacobs also explain how “[t]he beauty of direct quantum feedback, formulated by Wiseman and Milburn, is that it may be used to cancel the noise which drives the mean values of the dynamical variables”[6]. Their paper also shows how to “formulate, in a simple manner, feedback in linear quantum systems such that the best estimates of system variables are used to control the system”[6]. So, Doherty and Jacobs expand the realm of measurement feedback quantum control to regulate more quantum systems.

V. Conclusion

Quantum control is composed of open-loop quantum control and closed-loop quantum control. The former is derived relatively easily and has worked successfully in many instances. The latter can work on systems on which open-loop quantum control fails but also has to deal with the noise created from measurements. Many experiments have validated the efficacy of closed-loop quantum control.

One form of closed-loop quantum control is measurement feedback quantum control. This field has been proven to control linear quantum systems in addition to others. Two strategies of this field are Bayesian feedback and Markovian feedback. In theory, Bayesian feedback is better because it estimates the system's state using more previous states than Markovian feedback. Still, Markovian feedback may gain the advantage when the theories are put into practice due to its simpler design.

More approaches are still being developed and improved for quantum systems based on different approaches that are used in traditional control. Just as traditional control has expanded our technology and capabilities as a society beyond what humans could have imagined a few centuries ago, quantum control has the potential for drastically altering society. What is learned in the lab or is derived on paper impacts physical chemistry, atomic physics, and nuclear magnetic resonance research. While some quantum systems can already be controlled, more research will lead to improved control for more systems. Much work lies ahead but there is also great potential. Continual progress will eventually lead to great change.

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