Understanding Applied Nonlinear Elasticity Through Examples

Katrina Karouac, Raghu Echempati
Department of Mechanical Engineering
Kettering University
Flint, MI 48504
rechempa@kettering.edu

Abstract

Understanding the theory and principles of nonlinear elasticity or nonlinear mechanics is tough for students and also tough for practicing engineers. Although the governing equations in the traditional CAE (linear or nonlinear finite element) tools such as ANSYS, ABAQUS, LS-DYNA, SolidWorks, etc., use 1D or 2D or 3D linear/nonlinear elasticity formulations, students see little or no use to actually learn and understand the solution of the governing theoretical equations that use advanced numerical methods. The developers of CAE programs are obviously expected to deeply understand the assumptions made and the associated limitations in order to implement the solution processes to solve the theoretical equations. Offering more user-friendly CAE environment is still more challenging that involves experts from different fields of engineering and mathematical sciences. Research schools typically expect the students to write their own solution codes and validate those by using an appropriate CAE tool. What is the best method to teach a nonlinear elasticity course at a predominantly teaching school while also to demonstrate real life applications of the basic nonlinear theory? Automotive industries routinely use CAE tools to perform linear or nonlinear analysis of their very complex components and assemblies.

Nonlinearity arises due to geometry changes during deformation, material property changes, and changing loads (for example in dynamic environment). The purpose of this paper is to discuss the above-mentioned difficulties and limitations and also to provide few examples that demonstrate the nonlinear theoretical principles. The teaching and learning experiences of the author in delivering this course and also presenting invited talks on this topic at conferences will be discussed in detail in this paper.

KEY WORDS: Nonlinear elasticity, nonlinear finite element analysis, Assessment

Introduction

Understanding the basic concepts and distinctive characteristics of nonlinearity while a student has no real-life experiences can be challenging. Nonlinearity concepts can be split into three different categories: Geometric nonlinearity, material nonlinearity, and nonlinearity of the applied load, or a combination of all the three. Understanding each type of nonlinearity with examples can help expand the observations of nonlinearity in a students' perspective.

Nonlinearity should be applied every time that a linear relationship becomes questionable or invalid. Piece-wise linearization of a nonlinear behavior sometimes makes it easier which is what many math and CAE tools use. As a start for highly complex problems, sometimes linear FEA is often used before a nonlinear evaluation is attempted because the stiffness model is constant and solving process is quicker than that of a nonlinear analysis. The deflection for linear analysis is also much smaller than nonlinear counterpart because of the linear stress-strain curve for metals. An example of linear versus nonlinear stress-strain cures are shown in Figure 1 [1, 2]. A nonlinear analysis holds a nonlinear relationship between the applied forces or stress, and displacements or strain. Therefore, a different solving strategy is used for nonlinear problems, which are mostly numerical techniques. In the corporate world, nonlinear tools are used for companies to reduce cost, reduce lead times, use less material, and develop products that are better than their competitors.

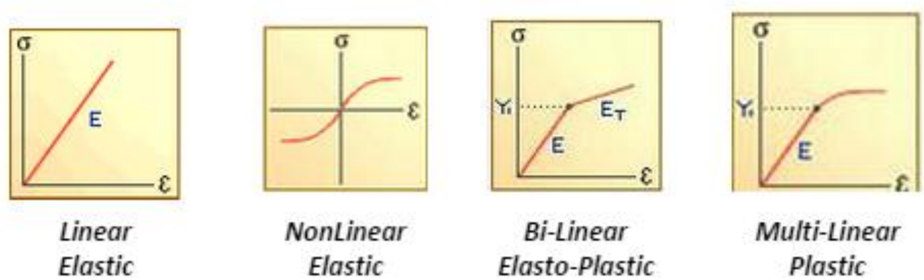

Figure 1. Simplified Stress-Strain Curves [2]

Nonlinear analysis comes into play when all three situations, geometrical, material, load/ contact, occur. Material nonlinearity is mostly in plastics, composite material, and glass, although for large deformation problems, metals show nonlinear behavior. Geometric nonlinearity is mostly buckling or when a large deformation occurs due to metal forming, which can ultimately change the stiffness. Load nonlinearity occurs when the applied load changes with time or displacement. This changes the stiffness due to deformation of the body. This is also in relation to contact nonlinearity in which the deformation of the body takes place. All these situations are related and can create a nonlinear analysis problem that can only solved numerically using a high-end CAE tool by performing finite element analysis.

A simple, everyday example of nonlinearity includes a paperclip. The bending and unbending of a paperclip is an example of an everyday item that we use which has material and geometrical properties of nonlinearity. When bending the paperclip, it demonstrates plastic stresses and residual stresses [3]. A few other simple examples include a staple, wooden bookshelf with books on a shelf, and pneumatic tires.

Literature Review

For this experimental study of nonlinearity with examples, many of the technical information included was from previous written papers of nonlinearity FEA. The example of a leaf spring was used, in part, because of the tremendous amount of previous work that has been conducted. Leaf springs are very popular in trucks due to the amount of load they can be pressed with. Mubea, as mentioned previously is one of the leading suppliers of leaf springs to OEM's. Their springs heavily represent material nonlinearity because of their glass fiber composite technology. Compared to traditional leaf springs, they have a progressive characteristic which allows only a single leaf spring. This reduces the unsprung mass associated with traditional leaf springs [6]. In the paper, Mubea chassis springs – development of GFRP leaf springs for best-in-class lightweight design and functional performance, it goes more in-depth of the production of the springs and the manufacturing process of the new spring [7].

The conventional design of leaf springs involves multiple leaves which shorten on the bottom of the structure. Specifically, Nonlinear Dynamics of Thick Leaf Spring Using Timoshenko Beam Functions was referenced to determine why exactly the leaf springs are considered nonlinear. This goes in depth about the shear deformation of the smaller length leaves. It also stresses the importance of modeling to find the optimized shapes and characteristics of the leaf spring [8].
In the paper, Performance Characteristics and Evaluation of Alternate Materials for Automobile Advanced Leaf Springs, by Stephen Takim, included some general background information on leaf springs and demonstrated the spring deformation pattern when under load. It also suggests that honeycomb structures can be made to withhold a high standard of load [9].

In, the paper on Design and Analysis of Composite Leaf Spring in Light Vehicle paper, the authors reference composite leaf spring material and analysis through ANSYS. The composite leaf springs have higher stiffness and higher natural frequency than steel leaf springs [10]. Fatigue behavior of glass fiber reinforced plastic (same material Mubea uses), is tested directly against the seven-leaf steel spring and is proven that it has a better manufacturing process as well as similar, if not better results for performance.

Material Nonlinearity

Simply put, material nonlinearity occurs when the relationship between stress and strain is no longer linear. Material nonlinearity is probably one of the most common and well-known nonlinearities. In nonlinearity, it is possible when deformation happens, material may not return to its original form. Most metals have a linear stress/ strain curve relationship and lower strain values. At higher strains, the material yields and the response becomes nonlinear and irreversible [1]. Typically, elastic materials relate to a linear behavior and plastic corresponds with a nonlinear behavior. Various types of plastic like hyper-elastic (rubber) can demonstrate this nonlinearity. Along with plastics, nylon, aluminum, and cast iron are common materials that are considered nonlinear [4]. In this case, to observe the characteristics of these materials, it requires an FEA program to calculate the structural response of the loads. In addition, one can introduce nonlinearities into the system by changing the temperature along with changing the stiffness of the material from loads.

An example can be through remembering the properties of ductile and brittle fractures from Materials class. Brittle fractures have no plastic deformation before fracture; they simply break. Ductile material goes through the necking process therefore executing signs of plastic deformation.

With that basic knowledge, material nonlinearity can be used in the design and analysis of leaf spring suspension in trucks. Based off a basic leaf spring design, two shackles on either side of the spring are bolted onto the frame of the truck and in the middle, bottom, is clipped onto the axle of the trucks through use of a center bolt of the spring. There are 3 fixed supports used. Dimensions are as follows in Table 1:

Table 1: Dimensions of Basic Leaf Spring Used

Leaves Used	8
Width (mm)	57
Thickness (mm)	8
Total Length (mm)	1000

To show how material can have a big effect on the stress and strain thus making it nonlinear, ANSYS CAE tool has been used as a demonstration to show this relationship. Structural steel was used to demonstrate a linear relationship and honeycomb material was used to show the nonlinear relationship. Honeycomb is a structure made up of hexagonal cells. Leaf spring manufacturers consider honeycomb material because of rigidity in shear, high toleration to stress, very durable, and low mass. The honeycomb gets its name because it considerably resembles that of a bee honeycomb [5].

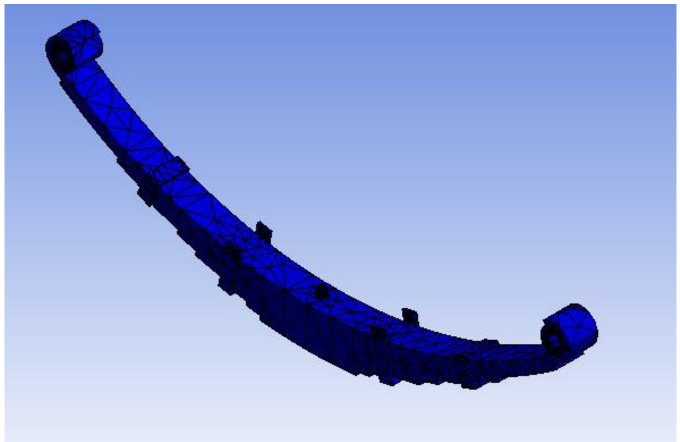

Figure 2. Leaf Spring at a time of 1 s and 0 N Force

When a force of 100,000 N was applied in the -Y direction over 4 seconds, the structural steel exhibited hardly any strain deformation.

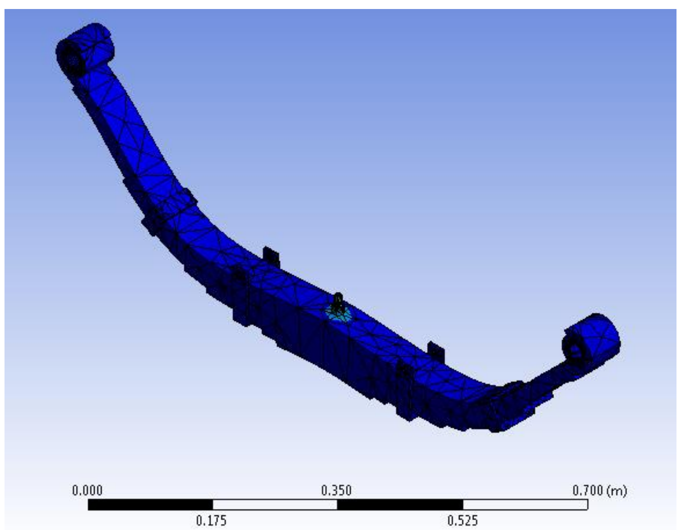

Figure 3. Strain Deformation of Steel Material at 10 s and 100,000 N Force

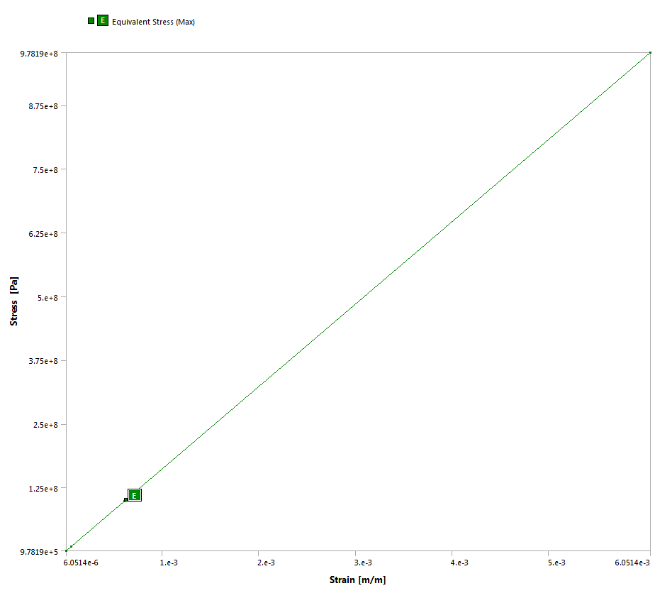

Figure 4. Stress-Strain Relationship of Steel Material

As we can see from Figures 3 and 4, steel exhibits a linear relationship when it comes to stress and strain. On the other hand, when the same force and time have been demonstrated with the honeycomb material, the stress-strain curve was nonlinear and observed more of a deformation in which it cannot go back to the original form.

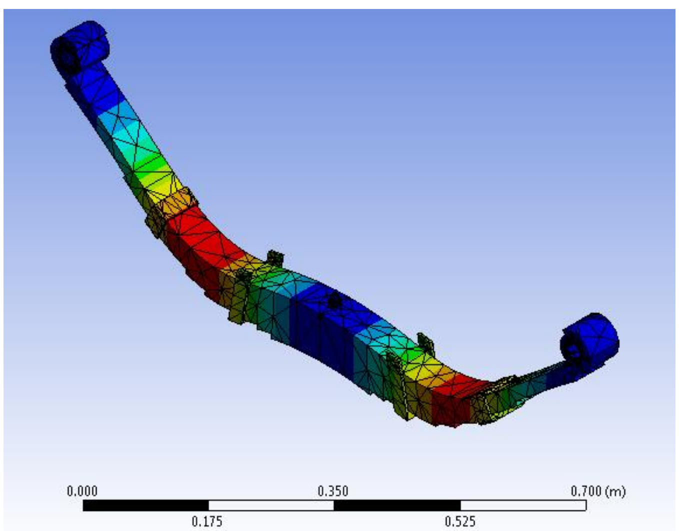

Figure 5. Strain Deformation of Honeycomb Material at Time 10 s and 100,000 N Force

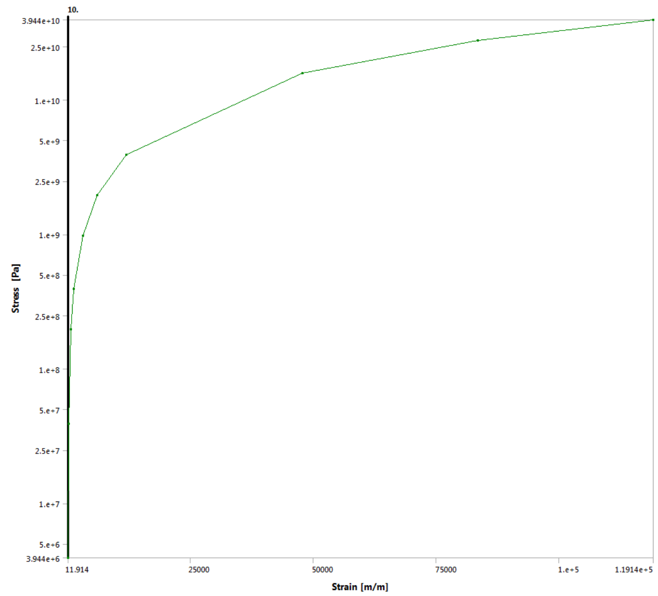

Figure 6. Stress-Strain Graph of Honeycomb Material

In addition to honeycomb material, one of the leaf spring leading suppliers, Mubea, has a fiber composite spring that is very common in today's trucks. This is a lightweight technology along with advantages for corrosion and damping properties [6 to 11]. With these lightweight springs, there is no pivot bearing and shackle required because of the single layer of material being used.

Geometric Nonlinearity

Geometric nonlinearity arises when a change of geometry occurs due to large deformation, large strain, or stress stiffening event. When you apply a load to something, it causes a large displacement and strain or rotation. When excessive deformation increases, it changes the stiffness of the object and it changes no matter what the material properties. Buckling is a common example that can occur due to an outside force. If the displacement is small, it is considered a linear model. If the displacement is larger, then it is considered a nonlinear case. However, if the case exhibits buckling, it is considered to be a nonlinear example because buckling's can happen in smaller displacements and can occur from axial and bending loads [2].

A simple example to understand is a fishing pole. If a fishing pole is held at 90 degrees from a person standing on a bridge, it is considered a linear case because there are no external components having any effect on the pole. Once a fish grabs onto the bait, the pole is still held at the 90 degree angle, however, it becomes a nonlinear situation because the fish is acting as large load at the end of the pole. With the weight of the fish on the end of the line, is creates a nonlinear condition. This demonstration can be shown in Figure 7.

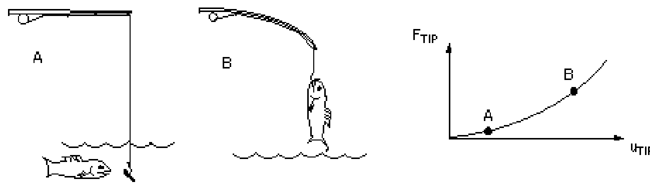

Figure 7. Fishing Pole Nonlinearity Example [11]

Following the truck leaf spring example from material nonlinearity, a phenomenon where the leaf spring can turn into a geometrical nonlinearity is when too much weight has been added to the vehicle and it exceeds the weight limit. This would be considered a large deformation in the leaf spring. This can be demonstrated by use of ANSYS and slowly applying load onto the leaf spring. Table 2 shows the input of time and the amount of force in the Y direction.

Table 2. Time Inputted versus Applied Load

Time (s)	Force applied in Y direction (N)
0	0
1	-100
2	-1000
3	-10000
4	-100000

When this data was inputted and a solution was generated, the stress-strain curve demonstrates that only a large load can create the geometry of the shaft to become nonlinear by deformation. At 100,000 N acting on the spring, the shaft began to deform. The following figures show the before and after of the 100,000 N force acting on the spring.

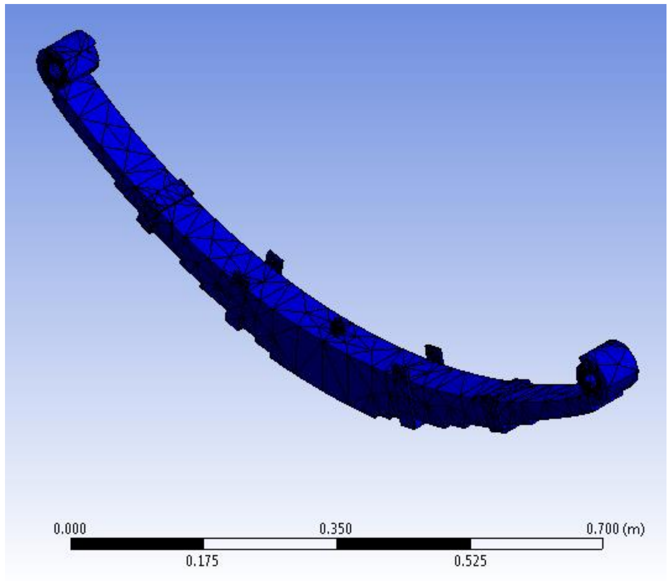

Figure 8. Leaf Spring at a Time of 1 s and a 0 N Force

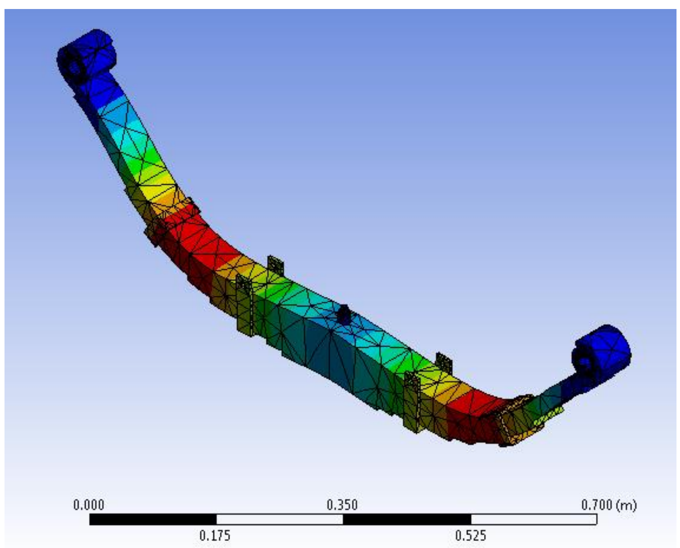

Figure 9. Leaf Spring at a Time of 4 s and a 100,000 N Force

To show this graphically, a stress-strain curve was created. Each point on the graph represents time and the last point on the graph represents the 100,000 N force on the spring. The stress increases as the strain increases making a nonlinear relationship.

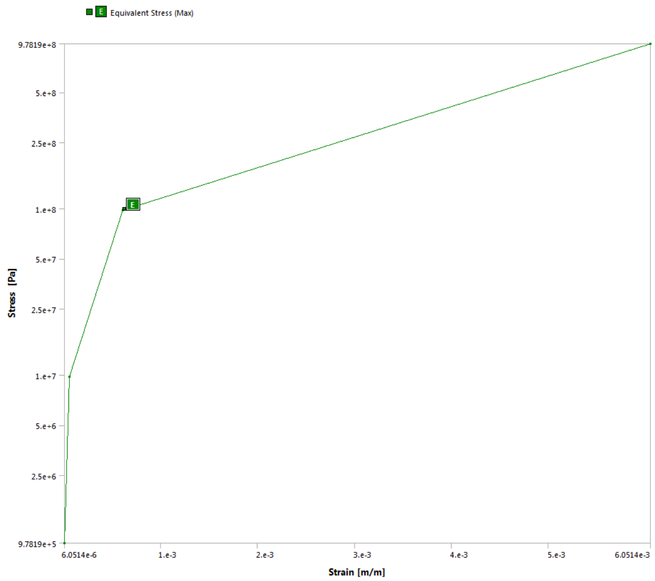

Figure 10. Trilinear Stress- Strain Graph for Leaf Spring

Load/ Contact Nonlinearity

Contact problems are important when considering metal forming, vehicle crashes, seal designs, brake systems, and gear systems [12]. Contact nonlinearities occur when structure boundary conditions change because of applied load [13]. Contact nonlinearity is commonly found when two objects clash or contact each other when a load is applied. Simply put, the diagram below shows a force and deflection plot when a beam hits a block. Right when the beam hits the block, the situation becomes nonlinear.

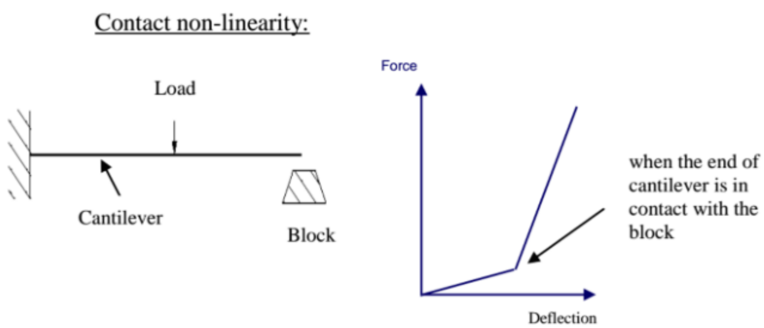

Figure 11. Contact Nonlinearity Example [13].

A common example of this is two gears touching or a vehicle crash. In a vehicle crash, the physical property of the vehicle changes due to the force and energy dissipated or absorbed during crash. Whether the crash is due to two or more vehicles impacting each other, or crash of a single vehicle hitting a pole, the structure of the vehicle(s) changes. This situation becomes a nonlinear problem.

To round up the leaf spring example, load nonlinearities can be demonstrated by changing loads over time due to rough road conditions. This can be found in both honeycomb (and composite) as well as structural steel. For example, over 10 seconds of time, there is an increasing load of 500,000 N in the negative Y direction. This increase is shown below in Table 3.

Table 3. Time Inputted Applied Load on Honeycomb Spring

Time (s)	Force applied in Y direction (N)
0	0
1	-100
2	-1000
3	-5000
4	-10000
5	-25000
6	-50000
7	-75000
8	-100000
9	-200000
10	-500000

For structural steel, the total deformation was 7.15E-004. For honeycomb material, the total deformation was 1.43E+005, which is very high. In this example, with that much time and load, the honeycomb material showed to be more nonlinear when it comes to contact and load.

Sheet metal forming example

As a final example of large deformation process, an example of a cylindrical sheet metal cup is presented in Figure 12. This shows real and virtual simulation of tearing behavior of the sheet due to excessive binder force that keeps the sheet metal in contact with the tools during its deformation. Large tensile loads develop due to punch (or die) travel causing the edges to tear and cause failure. Large deformation problems use the nonlinear or plastic deformation portion of the stress-strain curve. Usually, power law of plasticity is used for the plastic portion which is a function of plastic modulus, K and strain hardening exponent, n. The value of n for steel is usually 0.2 to 0.23 while for aluminum it is around 0.3. $\sigma = K\varepsilon^n$ is the stress-strain relationship for the plastic zone of the curve. Nonlinear solvers such as DYNA are used for this.

Figure 12. Failure of an example steel cup during drawing operation [14]

In real life, automotive sheet parts such as fenders require expensive tool sets that cost in excess of around $250,000 per set. Figure 13 shows an example of these [14]. In order to produce such complex geometries with less trash, nonlinear simulation tools need to be used such as LS-DYNA® or PamStamp®, etc. This will reduce the cycle time to market.

Figure 13. Example of an automotive fender and the die tool [14]

Summary/Conclusions

By using real-life examples, such as the leaf spring example and sheet metal parts, makes it easier for students to relate to the subject of nonlinearity. Breaking the characteristics of nonlinearity into material, geometry, and load, can help the student or the experienced engineer further understand the general meaning of nonlinearity. Material, geometry, and load all come together when considering if a component is linear or nonlinear.

Acknowledgements

Travel support provided by Kettering University to present this paper is acknowledged.

References

1. Material nonlinearity. (n.d.). Retrieved from https://abaqus-docs.mit.edu/2017/English/SIMACAEGSARefMap/simagsa-c-nlnmatnonlin.htm
2. Section 18: Nonlinear Materials. (n.d.). Retrieved from https://knowledge.autodesk.com/support/nastran-in-cad/learn-explore/caas/CloudHelp/cloudhelp/2017/ENU/NINCAD-SelfTraining/files/GUID-589E8565-42AF-497B-8FDD-541C954A01F0-htm.html
3. Gordon, L. (2013, April 01). Nonlinear FEA explained. Retrieved from https://www.machinedesign.com/news/nonlinear-fea-explained
4. *Material Non-linearity* (Rep.). (n.d.). Brunel. S. (2008). Understanding Nonlinear Analysis. Retrieved from https://nptel.ac.in/courses/105108072/mod07/hyperlink-4.pdf
5. Nonlinear shear modulus of re-entrant hexagonal honeycombs under large deformation. (2015, December 02). Retrieved from https://www.sciencedirect.com/science/article/pii/S0020768315004771
6. Ghosal, S., Sahoo, P., & Nath Saha, K. (n.d.). *NONLINEAR DYNAMICS OF THICK LEAF SPRING USING TIMOSHENKO BEAM FUNCTIONS* (Rep.).
7. New Identity AG. (n.d.). Fiber Composite Springs. Retrieved from https://www.mubea.com/products-technologies/automotive/suspension/fiber-composite-springs/
8. Asbeck, J., Dr., & Stimpfl, J., Dr. (2017). *Mubea chassis springs – development of GFRP leaf springs for best-in-class lightweight design and functional performance* (Tech.). doi:10.1007/978-3-658-18459-9_13
9. Takim, S. A. (2014). *Performance Characteristics and Evaluation of Alternate Materials for Automobile Advanced Leaf Springs* (4th ed., Vol. 11, IV, Tech. No. 2278-1684).
10. Khalokar, K. P., Alwat, S. J., & Chiwhane, D. S. (n.d.). *DESIGN AND ANALYSIS OF COMPOSITE LEAF SPRING IN LIGHT VEHICLE* (5th ed., Vol. 1, Tech. No. E-ISSN: 2394-8299). IJRISE.
11. R.V, N. (n.d.). FINITE ELEMENT STUDIES ON DEFORMATION CHARACTERISTICS OF NON-LINEAR MATERIALS IN QUASI-STATIC

REGIME. Retrieved 2018, from https://www.researchgate.net/figure/A-Fishing-Rod-Demonstrates-Geometric-Nonlinearity_fig1_291691367
12. *Week 10 - Lecture Nonlinear Structural Analysis*. (n.d.). Lecture. Retrieved from https://www.andrew.cmu.edu/course/24-688/handouts/Week 10 - Nonlinear Structural Analysis/Lecture Material/Week 10 - Nonlinear Structural Analysis - Lecture Presentation.pdf
13. How to distinguish primary and secondary beam in FEM formulation? (n.d.). Retrieved from https://engineering.stackexchange.com/questions/8922/how-to-distinguish-primary-and-secondary-beam-in-fem-formulation.
14. R. Echempati (2015): Lecture Notes of Metal Forming.