Smith Chart Use in RF Design and EMC Testing

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Abstract

When studied for the first time, the Smith chart can be very intimidating and perplexing. The study of it requires a proper background in the transmission line theory and the basics RF matching network design. This paper focuses on the selected background topics that naturally lead to the Smith chart construction. The application of the Smith chart is shown in several RF examples and ElectroMagnetic Compatibility (EMC) testing. These topics are related to a graduate student thesis and are based on the student research performed in the EMC laboratory at our university.

Introduction

The Smith Chart is a graphical tool for analyzing and designing transmission line circuits as well as designing RF impedance matching circuits without the need for detailed and tedious numerical calculations. It is also extensively used in EMC testing and performance verification. In this paper the fundamentals of the Smith Chart are explained applied to the network analyzer calibration procedure. Subsequently, the use of Smith chart in RF matching networks design is discussed. Examples include L networks used for matching the source to the load antenna.

This paper is organized as follows. Section 1 reviews the phenomenon of transmission line reflections at a load, which is the fundamental starting point in the study of the Smith chart. Section 2 shows the application of the developed theory in the network analyzer calibration procedure – perhaps the easiest and natural practical application of the Smith chart. In Section 3 a bit more advanced topics needed for the study of the Smith chart are explained, namely the resistance and reactance circles. These circles are utilized in Section 4 when explaining the RF matching networks design. Section 5 contains the summary and conclusions.

Section 1 Smith Chart – Load Reflection Coefficient

The Smith chart, shown in Fig. 1, is a graphical tool for analyzing and designing transmission line circuits as well as designing impedance matching circuits without the need for detailed and tedious numerical calculations, [1].

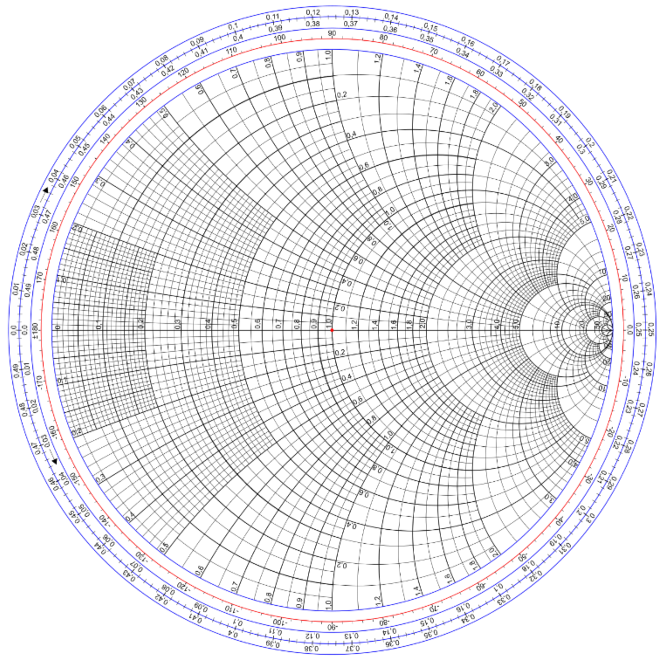

Figure 1: Smith chart

Smith chart is based on a polar plot of the load voltage reflection coefficient, $\hat{\Gamma}$. This reflection coefficient is obtained from the transmission line circuit shown in Fig. 2, [2], and is defined as

$$\hat{\Gamma} = \frac{\hat{Z}_L - Z_C}{\hat{Z}_L + Z_C} \tag{1}$$

where Z_C is the characteristic impedance of the transmission line.

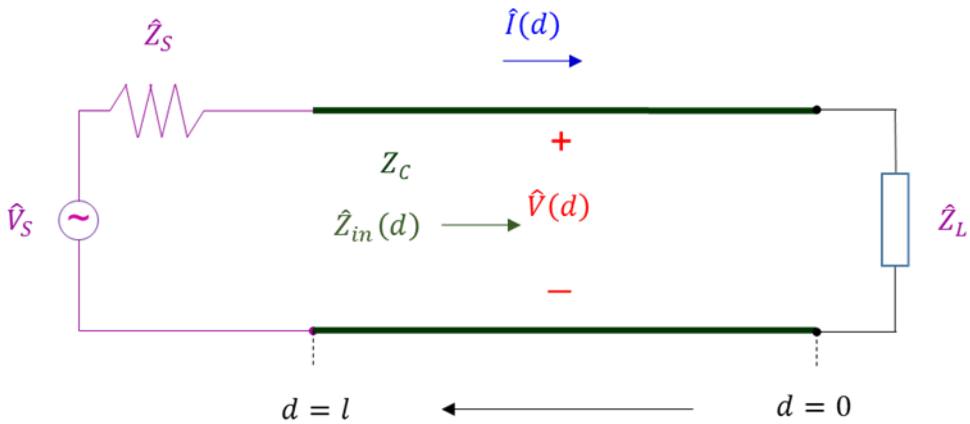

Figure 2: Transmission line driven by a source and terminated by a load $\hat{Z}_L$

Before we proceed with the Smith chart basics, let's discuss three special cases of the reflection coefficient in Eq. (1) when the line is terminated by a resistive load R_L, (these three cases are directly applicable to the network analyzer calibration procedure).

Short-Circuited Line $R_L = 0$ In this case the reflection coefficient is

$$\Gamma = \frac{R_L - Z_C}{R_L + Z_C} = \frac{0 - Z_C}{0 + Z_C} = -1 \tag{2}$$

Open-Circuited Line $R_L = \infty$ In this case the reflection coefficient is

$$\Gamma = \frac{R_L - Z_C}{R_L + Z_C} = \left. \frac{1 - \dfrac{Z_C}{R_L}}{1 + \dfrac{Z_C}{R_L}} \right|_{R_L \to \infty} = 1 \tag{3}$$

Matched Line $R_L = Z_C$ In this case the reflection coefficient is

$$\Gamma = \frac{Z_C - Z_C}{Z_C + Z_C} = 0 \tag{4}$$

In general, the reflection coefficient is a complex quantity, and as such it can be expressed as

$$\hat{\Gamma} = \Gamma e^{j\theta} = \Gamma_r + j\Gamma_i \tag{5}$$

Thus any reflection coefficient can be plotted as a unique point on the $\Gamma_r - \Gamma_i$ plane, as shown in Fig. 3.

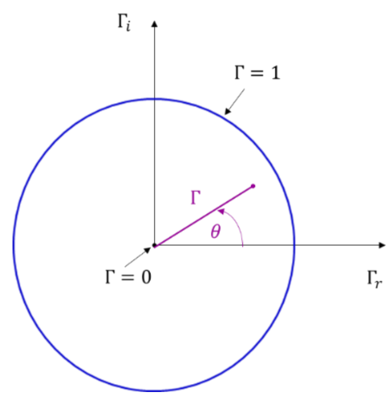

Figure 3: Unit circle on which the Smith chart is constructed

The magnitude, Γ, is plotted as a radius from the center of the chart, and the angle θ, $(-180° \leq \theta \leq 180°)$ is measured counterclockwise from the right-hand side of the horizontal Γ_r axis. Since each point on the Smith chart corresponds to a unique value of the voltage reflection coefficient at the load, the three special cases of the reflection coefficient discussed earlier (short, open matched load) correspond to the three special points shown in Fig. 4.

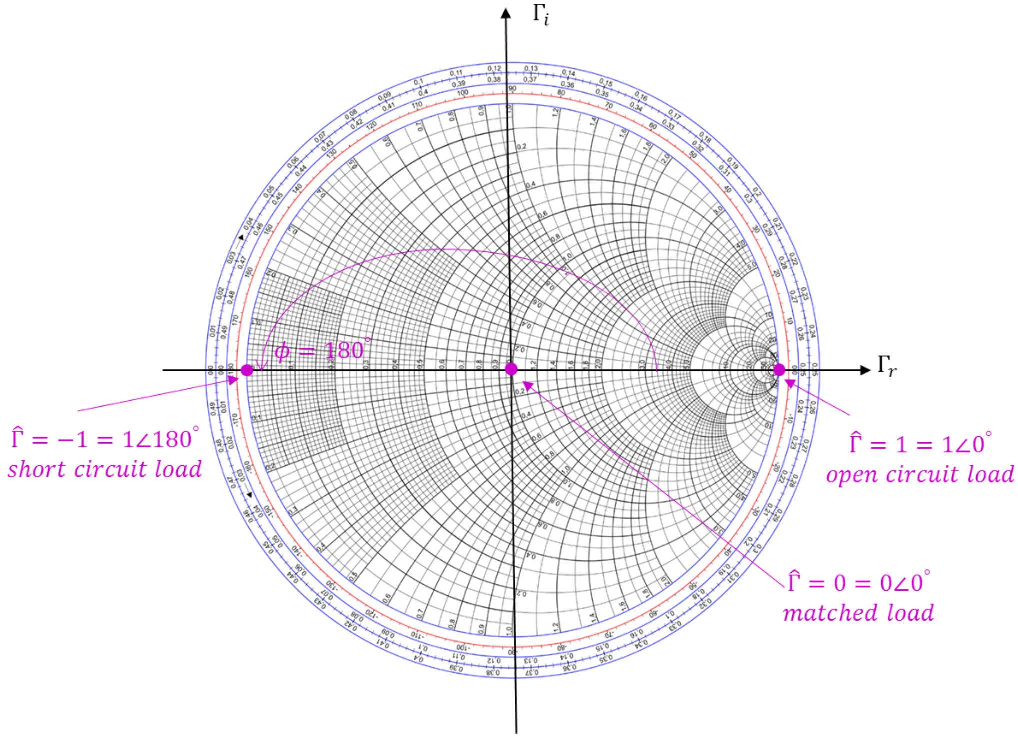

Figure 4: Reflection coefficient location for a short, open and matched load

We will refer to these point on the Smith Chart when discussing the network analyzer calibration procedure next.

Section 2 Network Analyzer Calibration Procedure

Perhaps the simpler practical application of the Smith chart is in the network analyzer calibration procedure. The network analyzer calibration procedure utilizes a calibration kit, like the ones shown in Fig. 5, which consists of a short, open, 50 Ω load attachment, and often a thru connector.

a) b)

Figure 5: a) N-type calibration kit, b) SMA-type calibration kit

A few different types of calibrations can be performed, depending on the parameter of interest [3]. If only the s_{11} measurements are required then the calibration is performed at port 1 with a short, open and 50 Ω (load) terminations as shown in in Fig. 6.

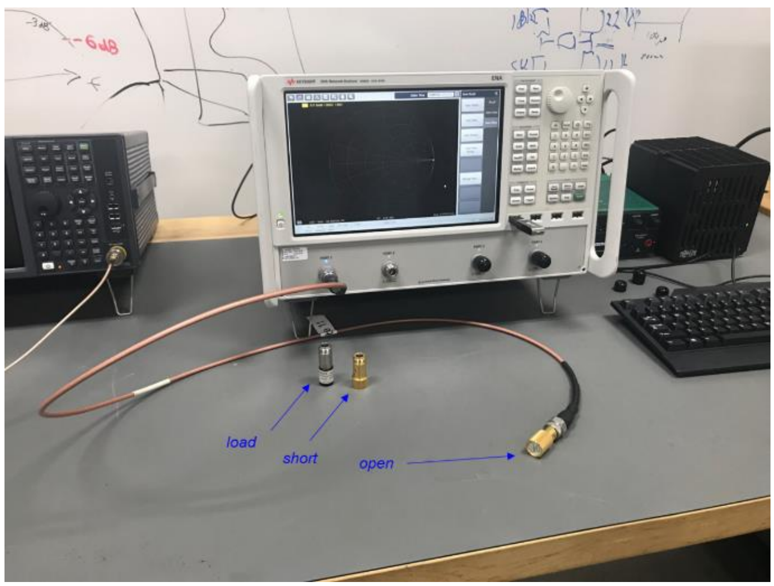

Figure 6: Calibration for s_{11} measurements

The results of the calibration can be verified using Smith chart menu of the network analyzer. When the calibration procedure is successful, the Smith chart plots of the voltage reflection coefficient should resemble the ones shown in Figures 7 through 9.

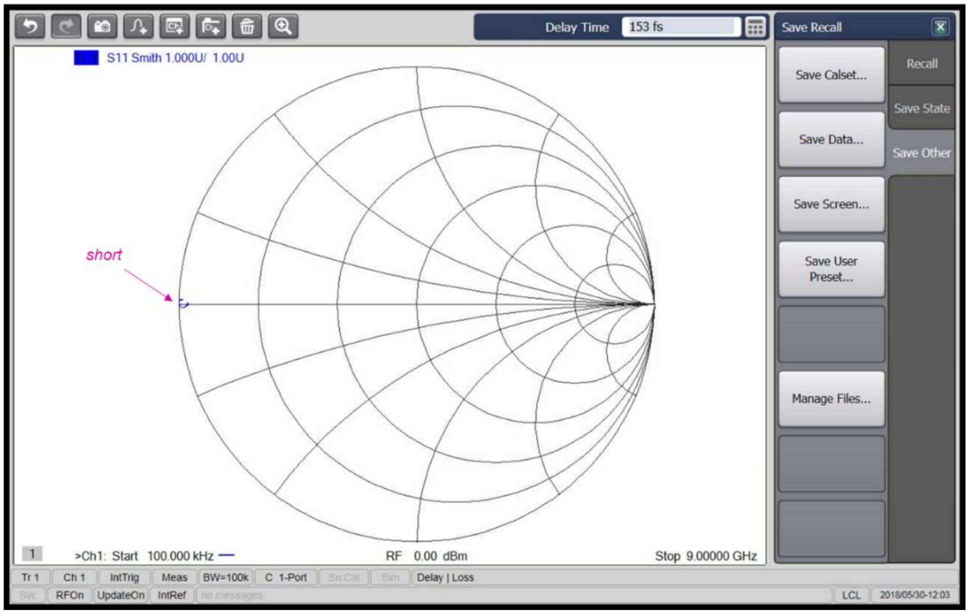

Figure 7: Calibration result for a short

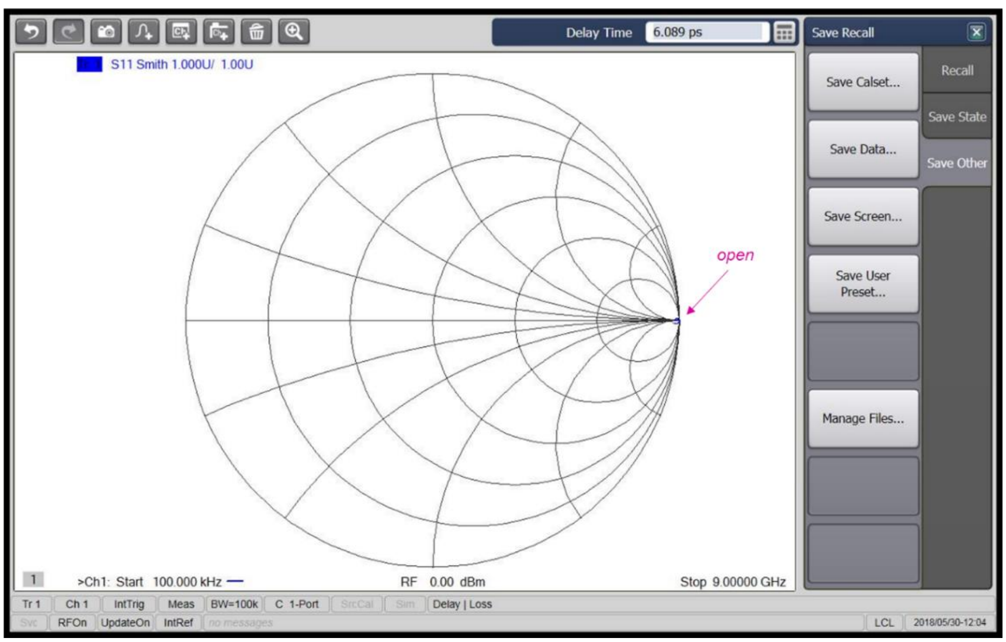

Figure 8: Calibration result for an open

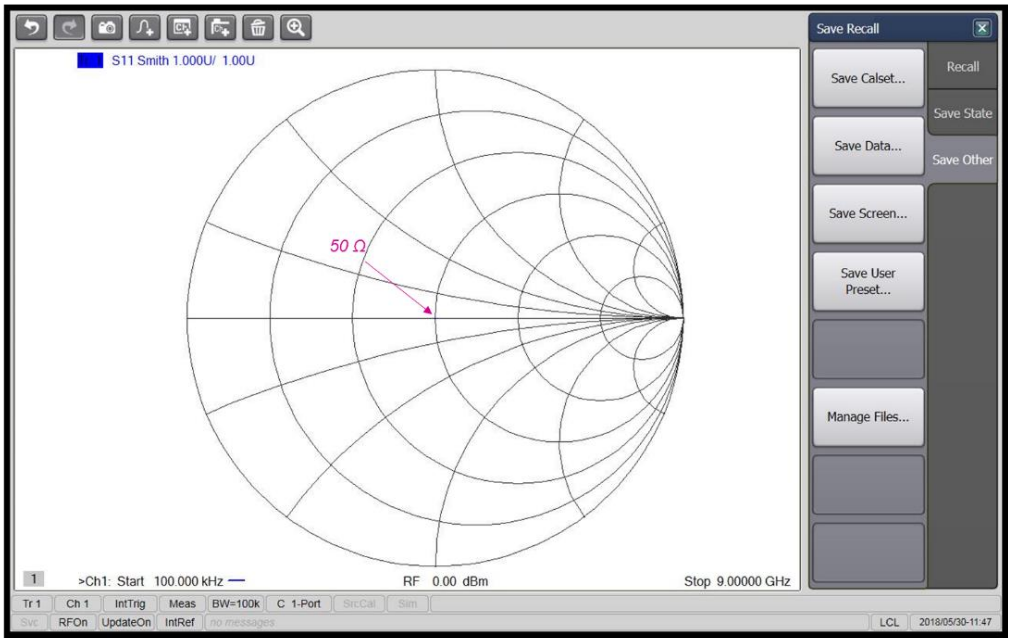

Figure 9: Calibration result for a (matched) load

Section 3 Smith Chart – Resistance and Reactance Circles

The real utility of the Smith chart lies in the fact that it can be used to convert from reflection coefficient to normalized impedances (or admittances) and vice versa. The normalization constant is usually the characteristic impedance of the transmission line. Thus, $\hat{z} = \hat{Z}/\hat{Z}_C$ represents the normalized version of the impedance $\hat{Z}$. This normalized impedance is a complex quantity with real and imaginary parts denoted as

$$\hat{z}_L = r_L + jx_L \tag{6}$$

and is related to the reflection coefficient by

$$\hat{z}_L = \frac{1+\hat{\Gamma}}{1-\hat{\Gamma}} = \frac{1+\Gamma_r + j\Gamma_i}{1-(\Gamma_r + j\Gamma_i)} \tag{7}$$

Combining Eqs. (6) and (7), after some manipulation produces two new equations:

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2 \tag{8a}$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2 \tag{8b}$$

Equations (8a) and (8b) describe the resistance and reactance circles, respectively. These circles can be plotted on the $\Gamma_r - \Gamma_i$ plane, as shown in Fig. 10.

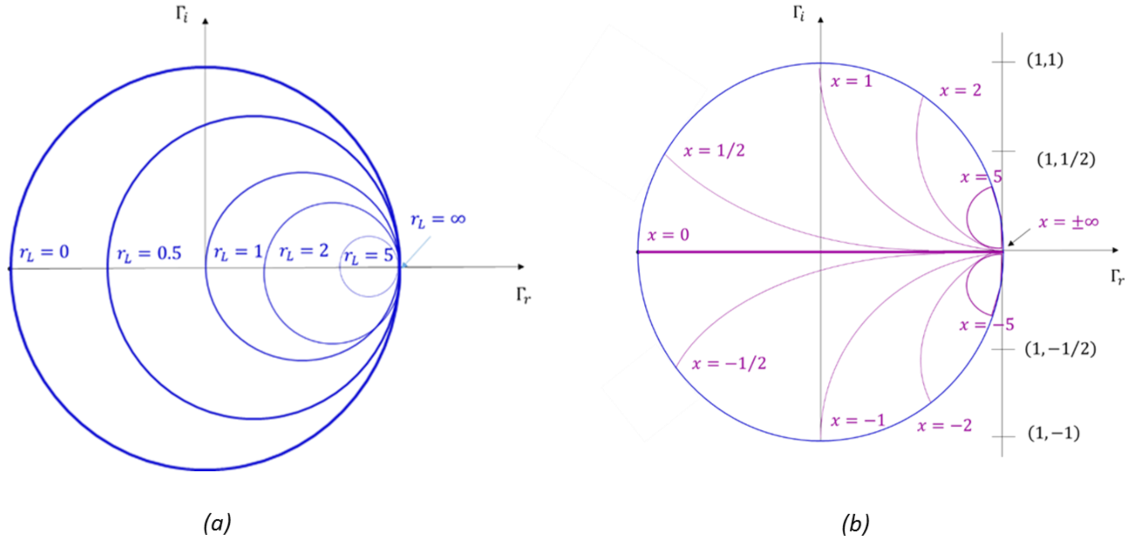

Figure 10: (a) Typical resistance circles, (b) typical reactance circles

When superimposed, the resistance and reactance circle produce the Smith Chart.

Section 4 RF Matching Networks

Smith chart found extensive use in designing RF matching networks used between the generator circuit and the load circuit. The load may be an antenna or any circuit with an equivalent impedance Z_L. When the transmission line is matched to the load, no reflections occur at the load and the power delivered to the load is a maximum.

The simplest solution to matching a load to a transmission line is to design the load circuit such that its impedance $Z_L = Z_0$. Unfortunately, this may not be possible in practice because the load circuit may have to satisfy other requirements. An alternative solution is to place an impedance matching network between the load and transmission line as shown in Fig. 11.

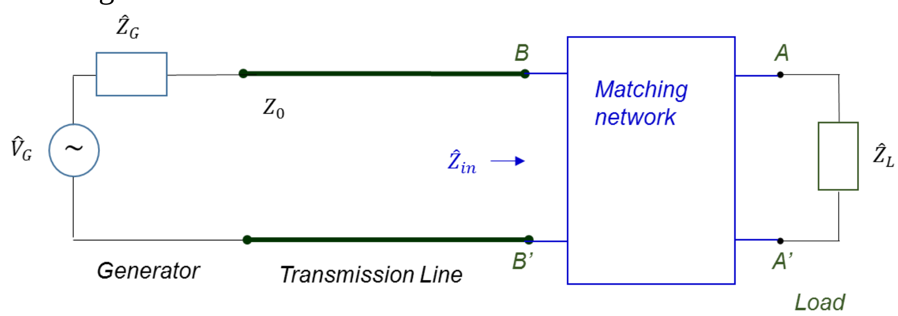

Figure 11: Matching network between the transmission line and load

The purpose of the matching network is to eliminate reflections at the juncture BB' between the transmission line and the network. This is achieved by designing the matching network to exhibit an impedance equal to Z_0 at BB' when looking into the network from the transmission line side.

Probably the simplest type of matching network is the *L-section*, which uses two reactive elements to match an arbitrary load impedance to a transmission line [4]. There are two possible configurations of this network, as shown in Fig. 12.

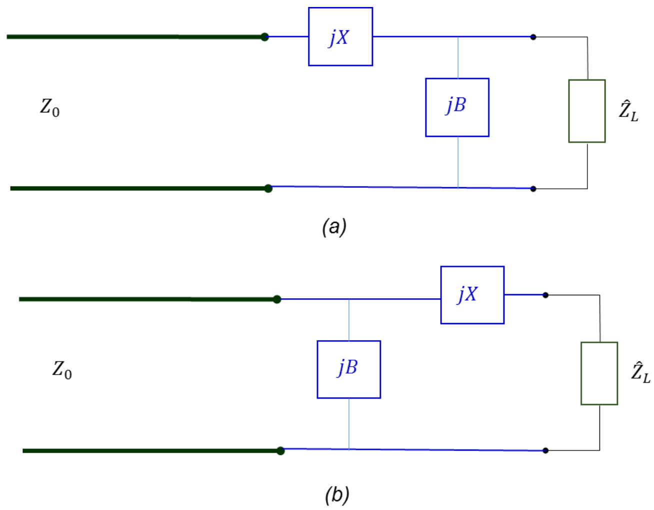

Figure 12: L-section matching networks (a) Network for $\hat{z}_L$ inside $1 + jx$ circle (b) Network for f $\hat{z}_L$ outside $1 + jx$ circle

In either of the configurations of Fig. 12, the reactive components may be either inductor or capacitors, depending on the load impedances. Thus, there are eight distinct possibilities for the matching circuit for various load impedances. In this paper, we will consider four of the possible eight configurations. The first two configurations (low-pass network), are shown in Fig. 13.

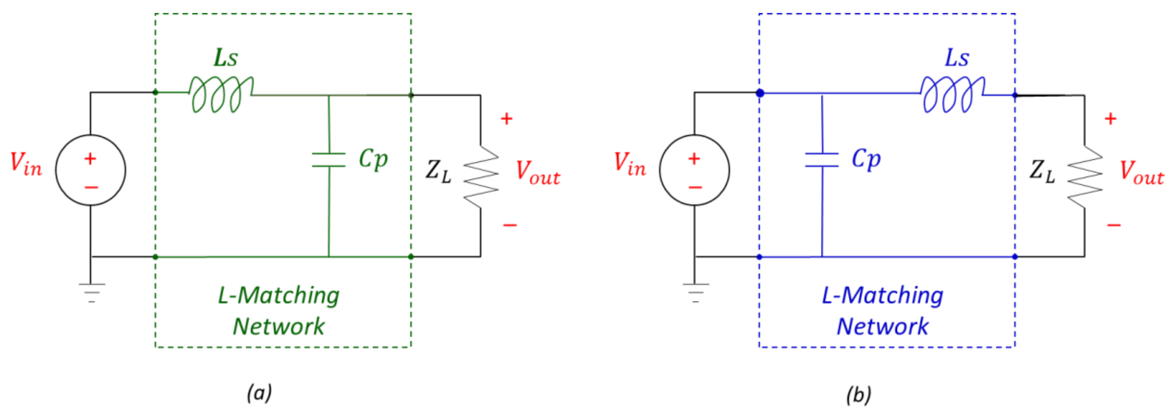

Figure 13: Low-pass matching network with a shunt capacitor: a) on the load side b) on the source side

These low-pass matching networks can be designed with using the Smith chart shown in Fig. 14. The red line in Fig. 14 shows the separation of the two possible regions of the *L* Network topologies shown in Figure 13.

The two green lines, labeled C_p and L_S correspond to the green matching network in Fig. 13(a). The parallel capacitor C_p moves the impedance on an admittance circle in clockwise direction until it crosses the impedance circle leading towards the center of the Smith chart. The series inductor L_s moves the impedance on the impedance circle clockwise towards the center of the Smith chart, i.e., the desired value of normalized impedance: 1+j0.

The two blue lines, labeled L_S and C_P correspond to the blue matching network in Fig. 13(b). Now, the series inductor L_S moves the impedance on an admittance circle in clockwise direction until it crosses the impedance circle leading towards the center of the Smith chart. The parallel capacitor C_P moves the impedance on the impedance circle clockwise towards the center of the Smith chart.

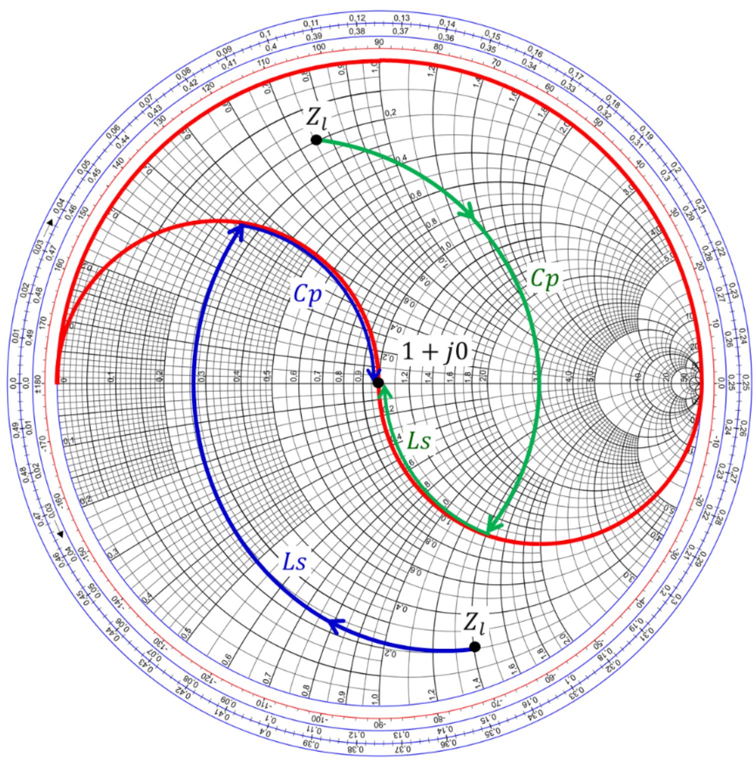

Figure 14: Smith Chart use for low-pass matching network

The second set of configurations (high-pass matching network), is shown in Fig. 15.

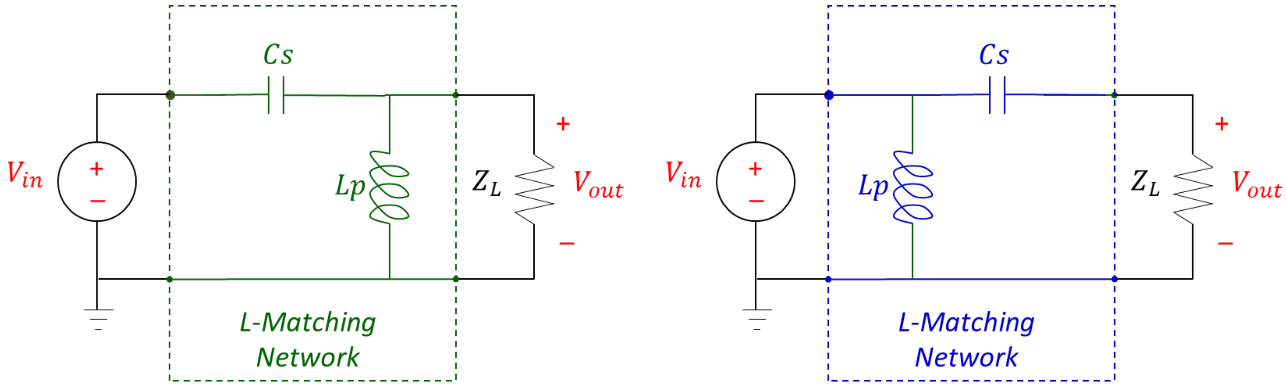

Figure 15: High-pass matching network with a shunt inductor: a) on the load side b) on the source side

Again, these high-pass matching networks can be designed with using the Smith chart shown in Fig. 16. The red line in Fig. 16 shows the separation of the two possible regions of the *L* Network topologies shown in Figure 15. The green lines correspond to the network shown in Fig. 15(a) while the blue lines correspond to the network shown in Fig. 15(b). The impedance point moves along these lines in a manner similar to that explained for the networks of Fig.13.

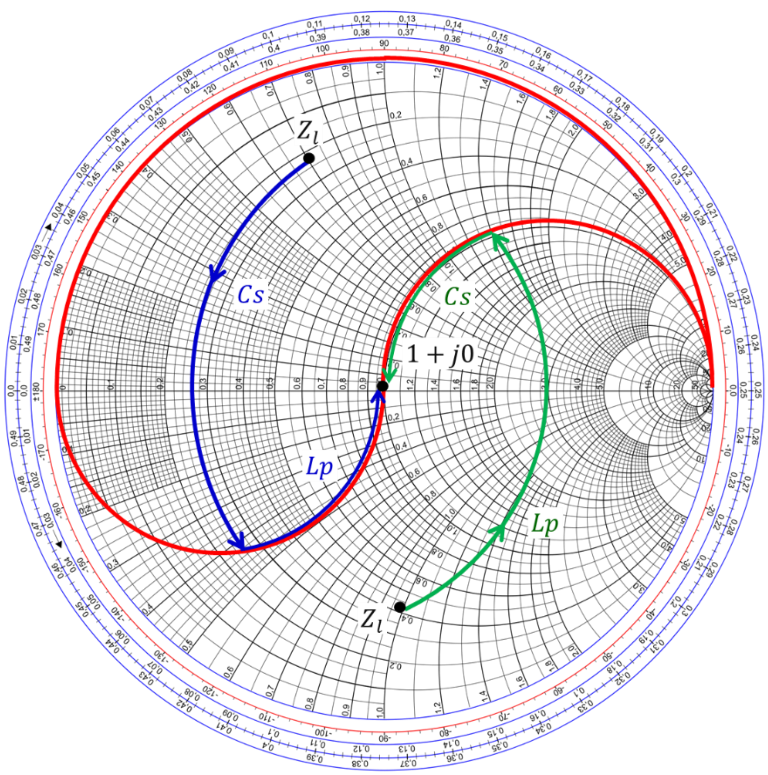

Figure 16: Smith chart use for high-pass matching network

Section 5 Summary and Conclusions

This paper presented two basic applications of the Smith chart in ElectroMagnetic Compatibility (EMC) testing and in designing RF impedance matching circuits. The EMC application focused on network analyzer calibration procedure while the RF examples showed the Smith chart use in the L-network matching circuit design. These two applications show the practicality of the Smith chart and serve as stepping stages in the explanation of the often intimidating topic when studied for the first time.

References

[1] Ulaby, T. U. and Ravaioli, U., *Fundamentals of Applied Electromagnetics*, 7th ed., Pearson, Upper Saddle River, NJ, 2015.

[2] Adamczyk, B., *Transmission Line Reflections at a Resistive Load*, In Compliance Magazine, January 2017.

[3] Adamczyk, B., Teune, J., *S-Parameter Tutorial – Part II: EMC Measurements and Testing*, In Compliance Magazine, September 2018.

[4] Ludwig, R. and Bogdanov, G., RF Circuit Design, 2nd ed., Pearson, Upper Saddle River, NJ, 2009.