The Value of Lumped Parameter Open Systems Momentum Analysis in Introductory Solid and Fluid Mechanics Courses

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Abstract
In virtually every introductory fluid mechanics book, the momentum principle (the combination of Newton's 2nd Law of Motion and Reynold's Transport Theorem) is introduced in distributed parameter integral form. This is not how the conservation of energy, conservation of mass nor the 2nd Law of Thermodynamics is introduced in the prior course in thermodynamics. Nor is the full lumped parameter momentum principle introduced in basic engineering mechanics except for special cases that result from that principle. The goal of this paper is to show the utility of such a formulation that would serve as a better bridge between thermodynamics, introductory solid mechanics and fluid mechanics courses.

I. Introduction

There is a disconnect in current educational pedagogy regarding the introduction of momentum analysis in the mechanics curriculum in general, from introductory thermo-fluid courses to dynamics courses. In classical mechanics courses, analysis typically begins with sum of forces or moments is zero in static situations. In dynamics, a lumped momentum equation, Newton's 2nd Law of Motion, is utilized but not in a form that can be used for open systems. In introductory fluid thermal courses, an integral formulation of the momentum principle is typically adopted early on, but not its lumped form. What is missing from the pedagogy is the possibility of utilizing a full lumped-form momentum expression that can be used in all mechanics courses, which would better bridge fundamental fluid-thermal concepts early and often, as will be shown in several examples in this paper.

It has become clear in years of teaching engineering subjects that there is a discontinuity between how related subjects are taught. By that, what is meant is that fundamental principles of mass, energy and momentum are indifferent to the application, yet are introduced and utilized very differently in various engineering courses. One good example of this disparity is the momentum principle, which is a combination of Newton's 2nd Law of Motion and Reynolds Transport Theorem. In every statics and dynamics textbook, for example, there is no mention of the momentum principle, even though, as will be shown in this paper, many problems (and entire chapters) can be best understood utilizing it.

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Most introductory statics and dynamics courses do teach various forms of "momentum methods." The plural form of "momentum methods" indicates that there is more than one, which can be observed in most statics and dynamics textbooks. Unfortunately, these methods are only special-cases of the more general momentum principle, but that is a fact unknown to prospective students. In fact, at no time in any statics and dynamics textbook that the author is aware of is the full momentum principle utilized.

The problem with not utilizing the full momentum principle is two-fold. First, there are many "momentum methods" introduced in textbooks unnecessarily overcomplicating a simple principle. Secondly, as a student progresses from one course to another, they virtually have to relearn what the momentum principle really is, almost as if it's something brand new.

It is the intent of this paper to show, through examples, that the same basic form of the momentum principle should be utilized throughout the curriculum, starting with basic statics and dynamics and progressing into the fluid and thermal sciences (fluid mechanics) and the rest of the curriculum.

II. Overview of Courses

The *Engineering Mechanics* course at Oakland University, ME 3200, mainly consists of sophomore-level undergraduate students. The course is a four-credit class, and involves both lecture and laboratory hands-on components. The lectures introduce new fundamental principles in the curriculum which most students will see as familiar (Newton's 2nd Law of Motion, energy, etc.). The lectures serve to introduce and apply these principles to problems in statics and dynamics. The laboratory component is strictly geared toward exploration, utilizing knowledge, analyzing experimental data, and comprehension learned in this very course.

The *Engineering Mechanics* course, previously called *Dynamics and Vibration*, was instituted in the 1970's to be the primary fundamental classical mechanics experience for introductory engineering students. As a four-credit course, the class meets twice a week for approximately an hour and a half. The lectures consist of a variety of introductions and plenty of example problems, plus collaborative student exercises. The lecture is thus broken up to include regular breakout sessions involving active learning techniques, student-centered learning and collaborative learning. Homework is assigned regularly to keep skills sharp and up-to-date.

The *Introduction to Fluid and Thermal Energy Transport* course at Oakland University, ME 3500, is essentially and introduction to fluid mechanics with some heat transfer with approximately a 60-40 split, respectively. The course is also a four-credit class, and involves both lecture and laboratory hands-on components.

III. Course Goals and Objectives

Oakland University catalogue course description for the Engineering Mechanics course:

"Statics and dynamics of particles and rigid bodies: analysis of structures, centroids and moments of inertia; kinematics, Newton's Second Law, work and energy, linear and angular impulse and momentum. With laboratory."

The course objectives, as determined by the departmental undergraduate committee, are as follows (parenthetical numbers represent the newest ABET outcomes):

1. Design and perform experiments. Analyze experimental data and write technical reports (1,3,4,5,6)
2. Apply Newton's second law to describe the static and dynamic equilibrium conditions of particles and rigid bodies. (1)
3. Determine the internal forces in the members of a truss. (1)
4. Express graphically and analytically the shear and bending moment of a beam. (1)
5. Apply Coulomb's law of dry friction to determine equilibrium forces on wedges and belts (1)
6. Determine the centroid of an area (1)
7. Determine the moment of inertia of a plane area with respect to a given axis. (1)
8. Determine the mass moment of inertia of a body about a given axis. (1)
9. Apply the kinematics theory to determine the relative velocity and relative acceleration between two points in a rigid body in planar motion (1)
10. Apply work and energy principles to describe and analyze the kinetics of a rigid body. (1)
11. Apply the principles of impulse and momentum to describe and analyze the kinetics of particles (1)

The major goal of this statics and dynamics course is to expose students to, and challenge them to think about, the entire taxonomy of the analysis process for a variety of systems either moving or stationary, open or closed. The lectures, including student-centered and active learning techniques, promote knowledge, comprehension and application. Regular homework and frequent small quizzes further promote these important aspects of the learning process. The hands-on laboratory experiences then take students through the analysis, data interpretation and evaluation.

Oakland University catalogue course description for the Introduction to Fluid and Thermal Energy Transport course:

"The fundamentals of fluid mechanics and heat transfer; fluid statics, conservation of mass and momentum; inviscid flow; internal viscous flow analysis; introduction to boundary layer theory; heat diffusion equation; dimensionless correlations of convection heat transfer, applications to engineering problems. With laboratory; includes experiment design."

The course objectives, as determined by the departmental undergraduate committee, are as follows (parenthetical numbers represent the newest ABET outcomes):

1. List and define elementary terminology related to fluid flow and heat transfer; explain the different flow classifications and flow field descriptions. Describe the different mechanisms of heat transfer and the different regimes of fluid flow. (1)
2. Design and perform experiments. Formulate, evaluate, and calculate experimental uncertainties of indirect measurements. Analyze experimental data and write quality technical reports. (3,4,5,6,7)
3. Explain the integral form of the conservation of mass and momentum principles and apply to a variety of static and dynamic fluid problems. (1)
4. Describe the development of the Bernoulli and Euler's equations for inviscid flows and list the underlying assumptions; apply the Bernoulli equation to appropriate engineering problems. (1)
5. Describe the meaning and the physical significance of the continuity and Navier-Stokes equations. (1)
6. Derive the velocity profile in a simple laminar viscous flow; explain viscous drag and the role of the Reynolds number in distinguishing between laminar and turbulent flows; evaluate the head loss and pressure drop in single-path piping systems. (1)
7. Use the heat diffusion equation with appropriate boundary conditions to determine a steady one-dimensional temperature distribution in a solid. (1)
8. Describe viscous and thermal boundary layers; select and apply suitable empirical convection correlations to determine the convective heat transfer coefficients for simple engineering geometries. (1)
9. Use convection correlations to evaluate the overall heat transfer coefficient in problems involving convection and conduction such as heat exchangers. (1)

IV. Momentum principle versus "momentum methods", and integral versus lumped-parameter formulations

Although Newton's 2nd Law of Motion is utilized throughout every statics and dynamics textbook, it has a major limitation – it is formulated for a particle, not for more general systems that are neither particles nor closed. For open systems where, in general, the mass of the system may change (as well as the velocity of the center of mass), due to mass transferred across system boundaries, Newton's 2nd Law is combined with Reynold's Transport Theorem [2]. The result is usually introduced in introductory fluid mechanics courses in integral form as follows:

$$\frac{d}{dt}\int_V \rho\vec{v}\,dV = \int_V \rho\vec{f}_b dV + \int_A \vec{f}_s\,dA - \int_A \rho\vec{v}(\vec{v}\cdot\vec{n})\,dA \qquad (1)$$

In equation 1, v is the velocity, ρ is the density, f_b the body force per unit mass, f_s the surface force per unit area. Equation 1 is generally referred to as either the conservation of momentum or the more appropriately named momentum principle. What is not seen in any available textbook is a *lumped parameter* formulation of this momentum principle, which should be stated in equation form as follows:

$$\frac{d\vec{M}}{dt} = \vec{F}_b + \vec{F}_s + \vec{F}_m \qquad (2)$$

There are three distinctly different external forces that may change the momentum of any system: surface forces (pressure, shear stress, etc.), body forces (gravity, electric and magnetic forces, buoyancy, etc.) and forces due to momentum transfer with mass transfer (thrust from a fluid jet, jet or rocket engines, etc.).

For closed systems, the last term is zero by virtue of no mass and therefore no momentum crossing the boundary of the system with it. In this case, the time rate of change of system momentum – the left-hand term which consists of the product of mass and velocity – becomes the familiar product of mass and acceleration as mass can be taken out of the derivative (it is constant by virtue of the conservation of mass principle). Thus, equation (2) reduces to:

$$M\vec{a} = \sum \vec{F}_b + \vec{F}_s = \sum \vec{F} \qquad (3)$$

This is the equation that is utilized in most if not all statics and dynamics textbooks when beginning dynamics problems whether open or closed, and essentially reverse engineering by reasoning some elements of (2). On the other hand, almost without exception fluid mechanics textbooks begin with equation (1) which is the integral form of (2). In both topical areas, equation (2) ought to be used at least at first. Doing so would bridge the areas of solid mechanics with the fluid and thermal sciences, which would be an advantage in teaching statics and dynamics since it is typically an earlier course in the curriculum. Conversely, utilizing the lumped parameter formulation of the momentum principle would give students in fluid mechanics an (ironically) deeper understanding of it, just like students introduced into thermodynamics exclusively utilize the lumped parameter formulation of the conservation of mass and energy principles.

V. Open system examples

There are many possible examples to utilize here proving out the idea that utilizing the lumped parameter formulation of the momentum principle is superior to the 'momentum methods' and such strewn throughout statics and dynamics books, but the most compelling case may be for open systems where mass is moving into and out of a system (either static as seen in many statics and dynamics textbooks, or moving as seen in many fluid mechanics textbooks).

Example #1:

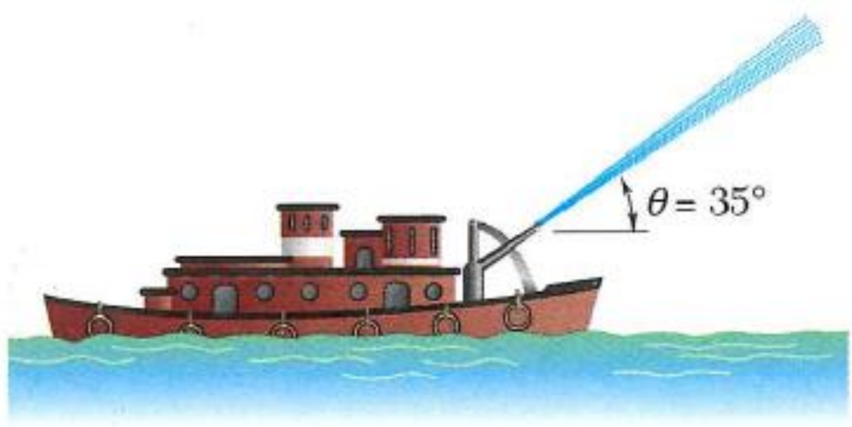


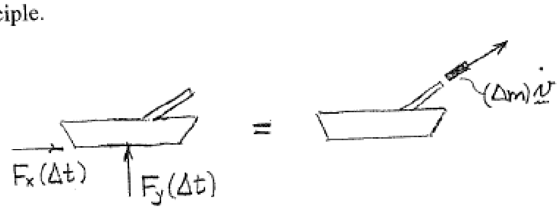
Figure 1: Problem 14.63 from the Beer and Johnston text [1]. "A hose discharges water at a rate of 8 m3.min with a velocity of 50 m/s from the bow of a fireboat. Determine the thrust developed by the engine to keep the fireboat in a stationary position."

How it's done in the solution manual [2]:

SOLUTION

Initial momentum: $(\Delta m)v_A = 0.$

Impulse-momentum principle.



$$(F_x\mathbf{i} + F_y\mathbf{j})(\Delta t) = (\Delta m)\mathbf{v}$$

$$F_x\mathbf{i} + F_y\mathbf{j} = \frac{\Delta m}{\Delta t}\mathbf{v} = \left(\frac{dm}{dt}\right)v\left(\cos 35^\circ\mathbf{i} + \sin 35^\circ\mathbf{j}\right)$$

x component:

$$\text{Engine thrust} = F_x = \frac{dm}{dt}v\cos 35^\circ$$

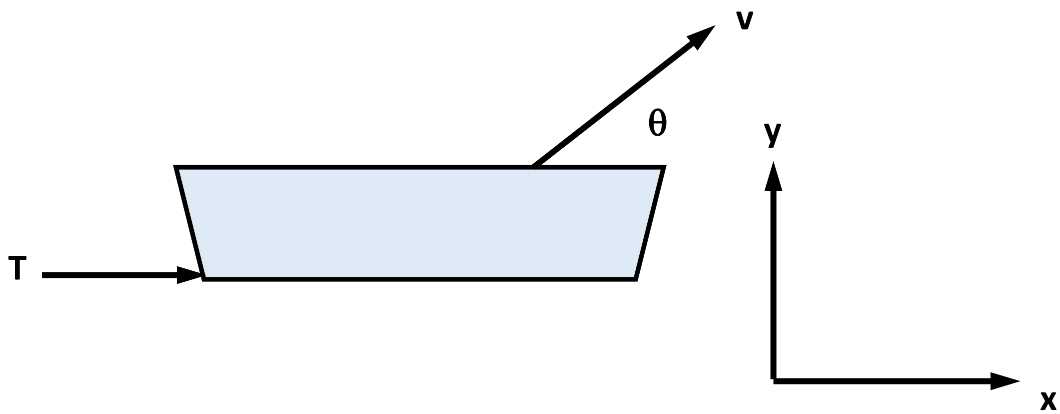
Data: $Q = 8\ \text{m}^3/\text{min} = \frac{8}{60}\ \text{m}^3/\text{s} \qquad \rho = 1000\ \text{kg/m}^3$

$$\frac{dm}{dt} = \rho Q = (1000)\left(\frac{8}{60}\right) = 133.333\ \text{kg/s}$$

$$F_x = (133.333)(50)\cos 35^\circ = 5461\ \text{N}$$

$F_x = 5.46$ kN ◀

There is nothing wrong with the way the authors above solved this problem. But the argument for the equation is what is termed “momentum methods” and puts together disparate pieces of information to form the final equation. Although correct, making such arguments problem after problem can be confusing to students. An alternate method for this problem then is to use the full momentum principle, equation (2). For this and every problem. To start, a full free-body diagram is drawn:



From equation (2) in the x-direction, it is noted that the system is static, so there is no inertia force (left hand side is zero), no body forces in the x-direction, and no surface forces. That only leaves the last term, of which there are two:

$$0 = \vec{F}_{m,x} = T - \dot{m}v_x \qquad (4)$$

The mass flowrate is negative since mass leaves the system (this is the scalar part of that term), while the x-component of velocity becomes positive vcosθ since that is the component in the x-direction. This results in

$$T = \rho\dot{V}v\cos\theta \qquad (5)$$

Equation (5) is the same as the result in the Beer and Johnston solution manual (the last few lines which include some unit conversions), but is arrived at by a different means – one that can be used for every single problem to avoid student confusion, and one that is simpler to arrive at.

Example #2:

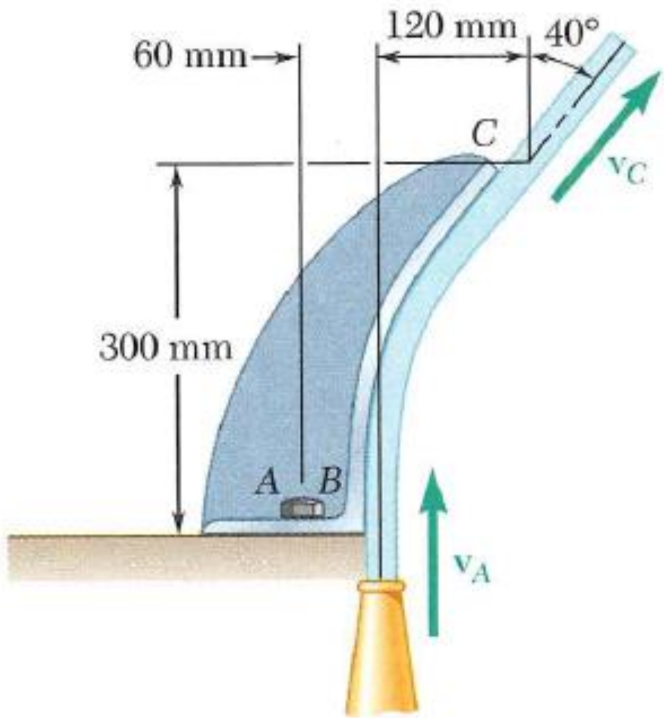
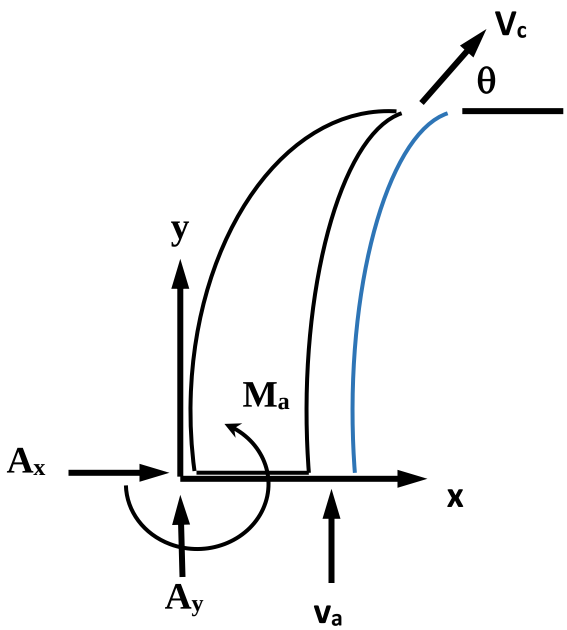


Figure 2: Problem 14.68 from the Beer and Johnston text [1]. "The nozzle shown discharges water at the rate of 800 L/min. Knowing that at both B and C the stream of water moves with a velocity of magnitude 30 m/s, and neglecting the weight of the vane, determine the force-couple system which must be applied to A to hold the vane in place."

For a symbolic analysis, the 60mm dimension above is taken as a, and thus the 120mm dimension is taken as 2a. The 300mm one is then 5a.

This results in the following FBD:



Since this is both a linear and angular momentum problem, it is important to center the origin where the potential point of rotation is to make the math easier (something that engenders critical thinking in students). For some background work, the velocity coming in, v_a, and leaving the system, v_c, can be expressed as

$$\vec{v}_a = v\vec{j} \tag{6}$$

$$\vec{v}_c = v\sin\theta\vec{j} + v\cos\theta\vec{j} \tag{7}$$

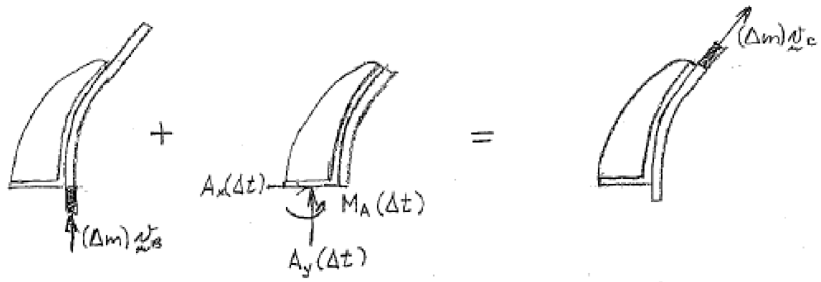
The Beer and Johnston solution for this problem [2] one again relies on "momentum methods", taking separate FBDs:

SOLUTION

$$Q = \frac{(800)}{60} = 13.333 \text{ L/s} \qquad \frac{dm}{dt} = \rho Q = (1.000 \text{ kg/L})(13.333 \text{ L/s}) = 13.333 \text{ kg/s}$$

$$\mathbf{v}_B = (30 \text{ m/s})\,\mathbf{j} \qquad \mathbf{v}_C = (30 \text{ m/s})(\sin 40°\mathbf{i} + \cos 40°\mathbf{j})$$

Apply the impulse - momentum principle.



$\xrightarrow{+}$ x components: $$0 + A_x(\Delta t) = (\Delta m)(30\sin 40°)$$

$$A_x = \frac{\Delta m}{\Delta t}(30\sin 40°) = (13.333)(30\sin 40°) \qquad A_x = 257\text{ N} \longrightarrow$$

$+\uparrow$ y components: $$(\Delta m)(30) + A_y(\Delta t) = (\Delta m)(30\cos 40°)$$

$$A_y = \frac{\Delta m}{\Delta t}(30\cos 40° - 30) = 13.333(30\cos 40° - 30)$$

$$= -93.6\text{ N}$$

$+\circlearrowright$ moments about A: $$(0.060)(\Delta m)(30) + M_A(\Delta t) \qquad A_y = 93.6\text{ N} \downarrow$$

$$= (0.180)(\Delta m)(30\cos 40°) - (0.300)(\Delta m)(30\sin 40°)$$

$$= 1.8(\Delta m) + M_A(\Delta t) - 1.6484(\Delta m)$$

$$M_A = \frac{\Delta m}{\Delta t}(-3.4484) = (13.333)(-3.4484) = -46.0\text{ N}\cdot\text{m}$$

$\mathbf{M}_A = 46.0\text{ N}\cdot\text{m} \circlearrowright$ ◀

$\mathbf{A} = 274\text{ N} \nwarrow 20.0°$ ◀

Alternately, using equation (2) and identifying the inertia term to be zero because it is a static system, and that there is no body force in the x-direction, that component of linear momentum becomes:

$$0 = A_x = T - \dot{m}_c v\sin\theta \tag{8}$$

And in the y-direction:

$$0 = A_y = T + \dot{m}_a v - \dot{m}_c v\cos\theta \quad (9)$$

For the problem of finding the reaction moment, equation (2) can used but with a position vector cross-product such that:

$$\vec{r} \times \left\{\frac{d\vec{M}}{dt} = \vec{F}_b + \vec{F}_s + \vec{F}_m\right\} \quad (10)$$

Here, the cross product gets distributed and the equation is all moments. This results in

$$\vec{0} = \vec{r} \times \vec{F_s} + \vec{r} \times \vec{F}_{m,a} + \vec{r} \times \vec{F}_{m,c} \quad (11)$$

Where

$$\vec{r} \times \vec{F}_{m,a} = a\vec{i} \times \dot{m}v\vec{j} \quad (12)$$

$$\vec{r} \times \vec{F}_{m,c} = (3a\vec{i} + 5a\vec{j}) \times -\dot{m}(v\sin\theta\vec{j} + v\cos\theta\vec{j}) \quad (13)$$

$$\vec{r} \times \vec{F}_s = M_a\vec{k} \quad (14)$$

Substituting equations (12) through (14) back into (11), and performing the vector cross-products yields results only in the z-direction, reducing the vector expression to a scalar one:

$$0 = M_a + a\dot{m}v + (3a\dot{m}v\cos\theta - 5a\dot{m}v\sin\theta) \quad (15)$$

This yields the same answer as the Beer and Johnston solution, but the analysis is linked back to the general momentum principle as can be applied to any and all problems in statics, dynamics, and fluid mechanics regardless of whether the system is open or closed.

An example of a typical open system problem is one where the supporting reaction forced need to be determined for a conveyor belt system with coal being dumped on, and then discharged, from said conveyor, or problem #14.69 in Beer et al [1] for which the solution below is found in [2].

The kinematics portion of the problem is similar to the way it is treated regardless of which approach is taken (although we make students derive the kinematic equations from scratch every time, but that is the subject of what should be another paper). The difference comes in when kinetics comes into play. From the Beer et al [2] solution manual:

Example #3

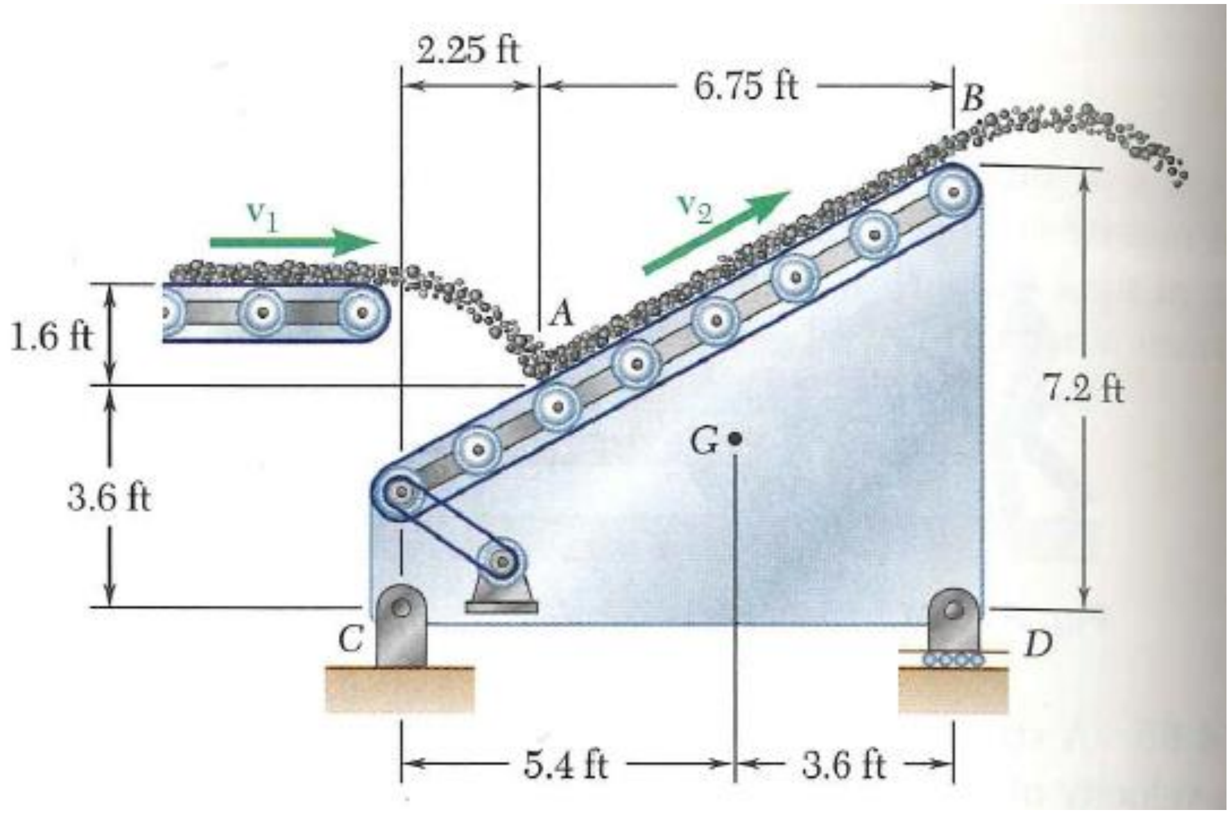
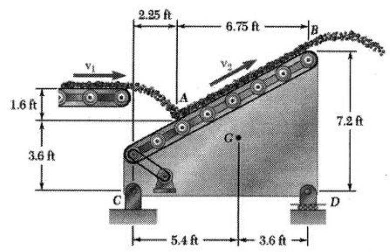


Figure 3: Problem 14.69 from the Beer and Johnston text [1]. "Coal is being discharged from a first conveyor belt at the rate of 240 lb/s. It is received at A by a second belt which discharges it again at B. Knowing that v1 = 9 ft/s and v2 = 12.25 ft/s and that the second belt assembly and the coal it supports have a total weight of 944 lb, determine the components of the reactions at C and D."

The above problem has the following solution [2]:



PROBLEM 14.69

Coal is being discharged from a first conveyor belt at the rate of 240 lb/s. It is received at A by a second belt which discharges it again at B. Knowing that $v_1 = 9$ ft/s and $v_2 = 12.25$ ft/s and that the second belt assembly and the coal it supports have a total weight of 944 lb, determine the components of the reactions at C and D.

SOLUTION

Velocity before impact at A:

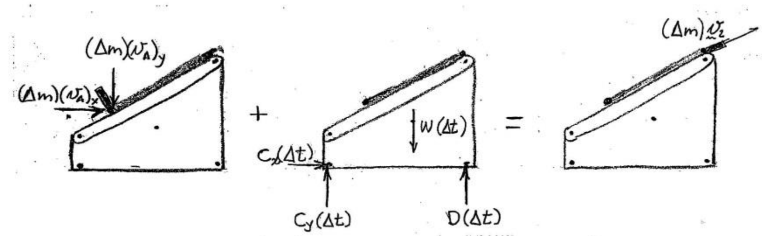
$$(v_A)_x = v_1 = 9 \text{ ft/s} \longrightarrow$$

$$(v_A)_y^2 = 2g(\Delta y) = (2)(32.2)(1.6) = 103.04 \text{ ft}^2/\text{s}^2 \qquad (v_A)_y = 10.151 \text{ ft/s} \downarrow$$

Slope of belt: $\tan\theta = \dfrac{7.2 - 3.6}{6.75}, \ \theta = 28.07°$

Velocity of coal leaving at B: $v_2 = 12.25(\cos\theta \mathbf{i} + \sin\theta \mathbf{j})$

Apply the impulse-momentum principle.



$\xrightarrow{+}$ x components: $(\Delta m)(v_A)_x + C_x(\Delta t) = (\Delta m)v_2\cos\theta$

$$C_x = \frac{\Delta m}{\Delta t}\left[v_2\cos\theta - (v_A)_x\right] = \frac{240}{32.2}(12.25\cos 28.07° - 9)$$

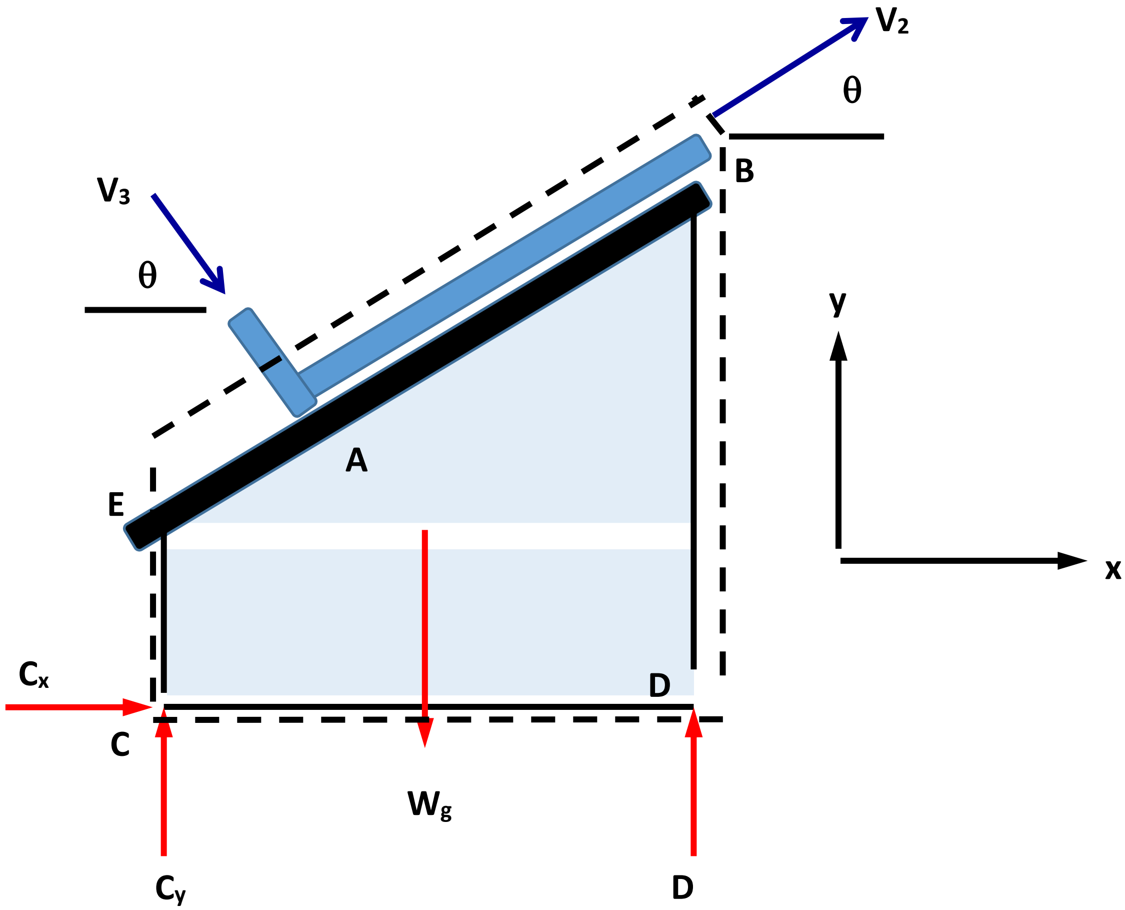
$$C_x = 13.48 \text{ lb}$$

+↻ moments about C:

$$(\Delta m)\left[-3.6(v_A)_x - 2.25(v_A)_y\right] + 9D(\Delta t) - 5.4W(\Delta t)$$

$$= (\Delta m)\left[-7.2\, v_2\cos\theta + 9v_2\sin\theta\right]$$

Alternately, a better way to perform the exact same problem as above would be to begin with eth full lumped-parameter formulation of the momentum principle, equation (2). Similarly to what is typically done with the vector form of Newton's 2nd Law of Motion, the momentum principle will be broken up into components and treated separately. The conveyor system as a whole is static and steady state (no property of the systems is changing with time); thus the left hand term which is the system's inertia, is zero no matter which component is being analyzed.



In the x-direction, the body force (gravity) has no component in this direction and is thus zero. The only terms remaining then are the surface force, which is C_x, and the force due to momentum transfer, $\vec{F}_m$. Since the force due to momentum transfer is eth momentum flowrate, or the product of mass flowrate and the velocity (average velocity in the case of the lumped parameter formulation), the x-component of the momentum principle becomes:

$$C_x = \dot{m}_2 v_2 cos\theta - \dot{m}_3 v_{3,x} \quad (16)$$

And since the mass flowartes are exactly equal as a consequence of the conservation of mass,

$$C_x = \dot{m}(v_2 cos\theta - v_{3,x}) \quad (17)$$

y-Direction.
In the x-direction, the body force (gravity) has its entirety in it, but is negative in relation to the defined coordinate system. The surface force in the y-component of C in addition to the force D at that roller support. And then the y-components of the force due to momentum transfer have to be accounted for. Thus, with the mass flowrates being equal,

$$C_y + D = W_{G+}\dot{m}(v_{3,y} - v_2\sin\theta) \quad (18)$$

Although there are enough parameter to determine Cx in equation (17), there are two unknowns in (18) and another equation is needed. There are no other non-trivial components of the momentum principle. Therefore, the angular form of the momentum principle, equation (10), must be used. The system is static and in steady state so the transient term is zero. Evaluating this about point C, it can be determined that:

$$\vec{r}_C \ x \left\{\frac{d\overrightarrow{M}}{dt} = \vec{F}_b + \vec{F}_s + \vec{F}_m\right\}_C = \vec{0}$$
$$= \vec{r}_{CG}x(-W_G)\vec{j} + \vec{r}_{CD}\ xD\vec{j} + \vec{r}_{CD}x\dot{m}\left(v_{3,x}\vec{i} - v_{3,y}\vec{j}\right) + \vec{r}_{CA}x\dot{m}(v_2\cos\theta\vec{i} - v_2\sin\theta\vec{j}) \quad (19)$$

The force C is not a variable in (19) and thus it can be solved for the unknown reaction force D at the roller support, which in turn can be substituted back into (18) to solve for C_y. Once C_y is obtained, a vector expression can be obtained for the reaction force C.

It can be seen that the above yields the exact same answer as the one obtained by Beer et al. However, we used the full lumped version of the momentum principle in the same way as it is used in the fluid and thermal sciences, and although our solution is longer, that by itself is not necessarily a bad thing and indeed may add to a student’s knowledge of this fundamental principle of nature. And that in the end is the whole point of the exercise – an understanding of basic principles throughout multiple courses.

VI. Conclusion
This paper detailed a method of using the momentum principle in lumped form that is common to engineering mechanics (statics and dynamics and in fact vibrations as well), as well as fluid mechanics. At this point in time, momentum is taught vastly differently in these different courses. Having a common foundation (in fact, it really is in concept but less so mathematically) would bridge the divide between these different courses and has the potential of alleviating confusion of students as the basis for application problems is simplified. Several examples were given in this paper. Ultimately, though, it would be interesting to compare students' abilities to solve such problems under the two different learning systems – fundamental as outlined in this paper versus conventional that can be found in the various textbooks. Of course, that would necessitate teaching two equivalent groups of students and likely by the same instructor. This

would have to be repeated several times for a large enough sampling size. The problem here also is that this approach would touch at least 3 courses in a typical mechanical engineering curriculum, and unless there was a commitment to try this fundamental approach in all of the courses, it would 'muddy the water' and make any statistics inconclusive. That being said, it would be an interesting trial and one that should be undertaken in the future.

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